15. Solve the following boundary value problem using the appropriate Green's function :

$u'' = t\ ;\ u(0) + u(1) = 0,\ u'(0) + u'(1) = 0.$

16. Determine the eigenvalues and eigenfunctions of Sturm-Liouville system :

$u'' + \lambda u = 0\ ;\ 0 \leq x \leq \pi,\ u(0) = u(\pi) = 0.$

17. Using Fourier transform to solve $u_t - u_{xx} = 0$; $u(0, t) = 0$, $u(y, t) = 0$, $u(x, 0) = 2x$, where $0 < x < 4$, $t > 0$.

18. (a) Find the Fourier transform of :

$$f(x) = \begin{cases} x, & |x| \leq a \\ 0, & |x| > a \end{cases}$$

(b) Establish the relationship between Fourier transform and Laplace transform.

19. (a) Find $L\left\{\dfrac{\sin at}{t}\right\}$.

(b) Using Laplace transform to solve :

$y'' + 6y' + 5y = e^{-t}\ ;\ y(0) = 0,\ y'(0) = 1.$

20. (a) Evaluate $L\{(\cos 2t - \cos 3t)/t\}$.

(b) Evaluate $L^{-1}\left\{\dfrac{2s + 1}{(s + 2)^2\,(s - 1)^2}\right\}$.

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2015

Time : 4 hours

Full Marks : 100

The questions are of equal value.

Answer any ***five*** *questions from each Group.*

Symbols used have their usual meanings.

Group – A

(Marks : 50)

(ADVANCED CALCULUS)

1. If all the first partial derivatives of f exist and are continuous in an open set D, then f itself is continuous in D.

2. Let $F(x, y, z) = 0$, then prove that :

$$\left[\frac{\delta z}{\delta y}\right]_{x=\text{const.}} \left[\frac{\delta y}{\delta x}\right]_{z=\text{const.}} \left[\frac{\delta x}{\delta z}\right]_{x=\text{const.}} = -1.$$

3. Let L be a linear transformation from R^n to R^m

represented by the matrix (ais). Then there is a constant B such that $|L(p)| \le B|p|$ for all points p.

4. Let S and T be given by :

$$S : \begin{cases} u = f(x, y, z) \\ v = g(x, y, z) \\ w = h(x, y, z) \end{cases} \quad T : \begin{cases} x = F(s, t) \\ y = G(s, t) \\ z = H(s, t) \end{cases}$$

Show that d(ST) = dS dT.

5. Let T(x, y) = (u, v), then show that :

$$\frac{\delta(u, v)}{\delta(x, y)} \cdot \frac{\delta(x, y)}{\delta(u, v)} = 1.$$

6. Let F be a function of three variables which is of class C′ in an open set D and let $p_0 = (x_0, y_0, z_0)$ be a point of D for which $F(p_0) = 0$. Suppose that $F_3(P_0) \neq 0$. Then, there is a function ϕ of class C′ in a nbd N of (x_0, y_0) such that $z = \phi(x, y)$ is a solution of F(x, y, z) = 0, for $(x, y) \in N$ and such that $\phi(x_0, y_0) = z_0$.

7. Define smoothly equivalent and show that if γ_1 and γ^2 are smoothly equivalent curves, then $L(\gamma_1) = L(\gamma_2)$.

8. Let Σ be a smooth surface and p be a point lying on Σ. Then the normal to Σ at p is orthogonal to any smooth curve which lies on Σ and passes through p.

9. If Σ_1 and Σ_2 are smoothly equivalent surfaces and w is a continuous 2 form defined on their trace. Then $\iint_{\Sigma_1} w = \iint_{\Sigma_2} w$.

10. Verify Green's theorem for w = xdx + yxdy with D as the unit square with opposite vertices at (0, 0) and (1, 1).

Group – B

(Marks : 50)

(PARTIAL DIFFERENTIAL EQUATIONS)

11. Find the general solution of the equation $3u_x + 2u_y = 0$.

12. Solve : $u_x = 2u_t + u$, where $u(x, 0) = 6e^{-3x}$.

13. Obtain the general solution of $u_{xx} + u_{xy} - u_{yy} = 0$.

14. Determine the solution of :

$$u_{xx} - u_{yy} = 1,$$
$$u(x, 0) = \sin x$$
$$u_y(x, 0) = x$$