5. (*a*) Define a planar graph. State and prove Euler's formula for planar graph.

OR

(*b*) Define a cutset in a graph. Prove that the ringsum of any two cutsets in a graph is either a third cutset or an edge-disjoint union of cutsets. Verify this by giving an example.

6. (*a*) (*i*) Obtain the sum of products and product of sums canonical forms of the expression $\left[(x_1+x_2)(x_3x_4)'\right]'$.

(*ii*) In any Boolean algebra, show that

(1) $a = 0 \Leftrightarrow ab' + a'b = b$

(2) $(a+b')\ (b+c')\ (c+a') = (a'+b)\ (b'+c)\ (c'+a)$.

OR

(*b*) (*i*) State and prove De Morgan's laws for a Boolean algebra.

(*ii*) Simplify the following Boolean expression and then construct the circuit diagram of reduced Boolean expression :

(1) $xy + xy' + x'y'$

(2) $xy'z + (z+y)\,x'$

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2016

MATHEMATICAL METHODS - I

Time : Three Hours] [*Maximum Marks* : 80

The figures in the right-hand margin indicate marks. Answer from both the Sections as directed.

SECTION-A

1. Answer any **four** of the following : 4×4

(*a*) State and prove change of scale property for Laplace transform of a function and hence evaluate $L\left[\frac{\sin at}{t}\right]$, given that $L\left[\frac{\sin t}{t}\right] = \tan^{-1}\frac{1}{s}$.

(*b*) Find a Fourier Series to represent x^2 in the interval $(-l, l)$.

(*c*) Find the Fourier transform of

$$f(x) = \begin{cases} 1, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$$

Hence evaluate $\int_0^\infty \frac{\sin x}{x}\,dx$.

(*d*) Find the Fourier Cosine transform of e^{-x^2}.

(*e*) Show that the maximum vertex connectivity one can achieve with a graph G of n vertices and e edges $(e \geq n-1)$, is the integral part of the number $\frac{2e}{n}$; that is $\left[\frac{2e}{n}\right]$.

(*f*) Draw the logic network diagram of $ab'+a'b$.

OR

2. Answer **all** the questions from the following : 2×8

(*a*) Find the Laplace transform of $2t\cos^2 t$.

(*b*) State initial value theorem of Laplace transform of a function.

(*c*) Define an odd function with an example.

(*d*) Define Fourier Cosine and Sine transform of a function.

(*e*) Show that the number of vertices of odd degree in a graph is always even.

(*f*) Define a Hamiltonian circuit in a graph.

(*g*) Define a Boolean homomorphism.

(*h*) Write the truth table for $(P \vee Q) \wedge \sim P$.

SECTION-B

Answer **all** questions of the following : 16×4

3. (*a*) (*i*) Evaluate $\int_0^\infty \frac{\cos at - \cos bt}{t}\, dt$

(*ii*) Find $L^{-1}\left[\frac{s}{(s^2+1)(s^4+4)(s^2+9)}\right]$

OR

(*b*) (*i*) Solve : $(D^3-3D^2+3D-1)y = t^2e^t$ given that $y(0)=1$, $y'(0)=0$, $y''(0)=-2$

(*ii*) Define a unit step function. Express the following function in terms of unit step function and find its Laplace transform.

$$f(t)=\begin{cases} 0 & , \quad 0<t<1 \\ t-1 & , \quad 1<t<2 \\ 1 & , \quad t>2 \end{cases}$$

4. (*a*) (*i*) Find a Fourier Series to represent $x-x^2$ from $x=-\pi$ to $x=\pi$.

(*ii*) Obtain the Fourier Sine transform of $e^{-|x|}$.

OR

(*b*) (*i*) Obtain Fourier Series for the function

$$f(x)=\begin{cases} \pi x & , \quad 0 \leq x \leq 1 \\ \pi(2-x) & , \quad 1 \leq x \leq 2 \end{cases}$$

Deduce that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots\ldots=\frac{\pi^2}{8}$.

(*ii*) If the initial temperature of an infinite bar is given by :

$$\theta(x)=\begin{cases} \theta_0 & , \quad |x|<a \\ 0 & , \quad |x|>a \end{cases}$$

determine the temperature at any point x and at any instant t, using Fourier transform.