MA/M. Sc.(IIS) —
Math (204)

2016

Time : 3 hours

Full Marks : 80

The figures in the right-hand margin indicate marks.

Answer from both the Sections as directed.

(ADVANCED COMPLEX ANALYSIS)

Section – A

1. Answer any **four** of the following : 4×4 = 16

(a) Determine the singularity of the functions :

(i) $\dfrac{1 - \cos z}{z}$

(ii) $\dfrac{z}{e^z - 1}$

(b) Evaluate :

$$\int_0^\infty y^3 e^{-2y}\, dy$$

(c) Evaluate $\oint_C z^{-2}\,dz$ around the circles $|z-1| = 1$.

(d) Investigate the convergence of the infinite product $\prod_{k=1}^{\infty}\left(1+\frac{1}{k^3}\right)$.

(e) Prove that $\Gamma(z+1) = z\Gamma(z)$.

(f) Find a bilinear transformation which maps points $z = 0, -i, -1$ into $w = i, 1, 0$ respectively.

OR

2. Answer **all** questions from the following :

2×8 = 16

(a) Find the principal value of i^i.

(b) Find the value of

$$\left(1-\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\ldots\ldots\ldots\ldots$$

(c) State Weierstrass theorem for infinite products.

(d) Evaluate $\Gamma\left(-\frac{7}{2}\right)$.

(e) Is sin z is bounded ? Justify your answer.

(f) Evaluate $\oint_C \frac{dz}{(z-a)^n}$, $n = 2, 3, 4, \ldots\ldots\ldots\ldots$, where $z = a$ is inside the simple closed curve C.

(g) Find the poles and residues of cot z.

(h) Define entire function of fractional order.

Section – B

Answer **all** questions of the following :

16×4 = 64

3 (a) Show that $\int_0^{2\pi}\frac{d\theta}{a+b\sin\theta} = \frac{2\pi}{\sqrt{a^2-b^2}}$ if $a > |b|$.

OR

(b) If f(z) is analytic in Ω, then $\int_C f(z)dz = 0$ for every cycle C which is homologous to zero in Ω.

4. (a) Find the partial fraction development of $\frac{1}{\cos \pi z}$ and show that it leads to

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots\dots\dots$$

OR

(b) Expand $f(z) = \frac{1}{(z+1)(z+3}$ in a Laurent series valid for (i) $1 < |z| < 3$ and (ii) $|z| > 3$.

5. (a) Prove that :

$$\xi(s) = 2^s \pi^{s-1} \sin\left(\frac{\pi s}{2}\right)\Gamma(1-s)\xi(1-s)$$

OR

(b) Prove that every entire function has a singularity at infinity.

6. (a) Prove that the zeros $a_1, \dots\dots\dots a_n$ and poles $b_1, \dots\dots\dots b_n$ of an elliptic function satisfy $a_1 + \dots\dots\dots + a_n \equiv b_1 + \dots\dots\dots + b_n$ (mod M).

OR

(b) Prove that :

$$p(2z) = \frac{1}{4}\left(\frac{p''(z)}{p'(z)}\right)^2 - 2p(z)$$