2016

Time : 3 hours

Full Marks : 80

The figures in the right-hand margin indicate marks.

Answer from both the Groups as directed.

(ADVANCED CALCULUS)

Group – A

1. Answer any **four** of the following : 4×4 = 16

(a) If $f(x, y) = \begin{cases} \dfrac{xy(x^2 - y^2)}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$,

then show that $f_{xy}(0, 0) = f_{yx}(0, 0)$.

(b) If $x = \phi(t)$, $y = \psi(t)$ and $z = f(x, y)$ where ϕ, ψ and f are differentiable, show that :

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$$

(c) If $x = r \sin\theta \cdot \cos\phi$, $y = r\sin\theta \sin\phi$, $z = r\cos\theta$ then show that $\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)} = r^2 \sin\theta$.

(d) Expand $x^4 + x^2y^2 - y^4$ about the point (1, 1) up to the terms of the second degree. Find the remainder R_2.

(e) Compute $\int_c xydx$ along the arc of the parabola $x = y^2$ from (1, –1) to (1, 1).

OR

2. Answer all questions from the following :

2×8 = 16

(a) If $z = x^3 - xy + y^3$, $x = r\cos\theta$, $y = r\sin\theta$, find $\frac{\partial z}{\partial r}$.

(b) If $u = f(x + 2y) + g(x - 2y)$, then show that :

$$4\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial y^2}$$

(c) If $z = e^{xy^2}$, $x = t\cos t$, $y = t\sin t$, compute $\frac{dz}{dt}$.

(d) Show that $z = \log\{(x-a)^2 + (y-b)^2\}$, satisfies $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$ for $(x, y) \neq (a, b)$.

(e) If $u = a\cosh x \cos y$, $v = a \sin hx \cdot \sin y$, find $\frac{\partial(u, v)}{\partial(x, y)}$.

(f) Verify existence and uniqueness of implicit function near the point (1, – 1) in case of $y^2 + 2x^2y + x^5 = 0$.

(g) Find $\iint_R (x + 2y)dxdy$ when R = [1, 2 ; 3, 5].

(h) Reduce $\int_c (1 - x^2)y\,dx + (1 + y^2)x\,dy$ where C is $x^2 + y^2 = a^2$, to double integral using Green's theorem.

Group – B

Answer all questions of the following : 16×4 = 64

3 (a) Show that the function | x | + | y | is continuous but not differentiable at origin.

OR

(b) State and prove Taylor's theorem for function of two variables.

4. (a) If v is a function of two variables x and y and $x = r\cos\theta$ and $y = r\sin\theta$, prove that :

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = \frac{\partial^2 v}{\partial r^2} + \frac{1}{r^2}\frac{\partial v}{\partial \theta^2} + \frac{1}{r}\frac{\partial v}{\partial r}$$

OR

(b) Prove that $\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}$ is invariant for change of rectangular axes.

5. (a) Find the maximum and minimum values of $x^2 + y^2 + z^2$ subject to the conditions $\frac{x^2}{4} + \frac{y^2}{5} + \frac{z^2}{25} = 1$ and $z = x + y$.

OR

(b) If $xyz = abc$, then show that the minimum value of $bcx + cay + abz$ is $3abc$.

6. (a) Prove :

$$\iint_R x^{p-1} y^{q-1}\, dxdy = \frac{\Gamma\left(\frac{p}{2}\right)\Gamma\left(\frac{q}{2}\right)}{\Gamma\left(\frac{p}{2}+\frac{q}{2}+1\right)}$$

OR

(b) Evaluate :

$$\int_0^\infty e^{-x^2}\, dx \ldots\ldots$$