6. State and prove Uryshon's Metrization Theorem.

OR

(b) State and prove Uryshon's Lemma.

2016

(January)

Time : 3 hours

Full Marks : 80

The figures in the right-hand margin indicate marks.

Answer from both the Sections as directed.

The symbols used have their usual meanings

(TOPOLOGY)

Section – A

1. Answer any **four** of the following : $4 \times 4 = 16$

   (a) Define topological space. Prove that intersection of any families of topologies for a set is a topology for the set.

   (b) Define the following terms. Discrete topology, interior of a set, base for a topology, Induced topology.

(c) Prove that every closed subset of a compact set is compact.

(d) If C is a connected subset of a topological space $(X, \Im)$ which has separation $X = A|B$, then prove that either $C \subset A$ or $C \subset B$.

(e) Prove that in a Hausdorff space a convergent sequence has unique limit.

(f) Define first axiom space. Prove that the property of being a first axiom space is a topological property.

OR

2. Answer all questions from the following :

$2 \times 8 = 16$

(a) Let $X = \{a, b, c\}$ and $\Im = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$. Find $d(\{a\})$.

(b) State the Kuratowski closure axioms.

(c) State the interior axioms.

(d) Is it true that locally connected sets are connected ? Justify your answer.

(e) Is it true that union of a family of connected sets is connected ? Justify your answer.

(f) Give example of T_0 space which is not a T_1 space.

(g) Define hereditary property. Give example of one heriditary property.

(h) State a necessary and sufficient condition for the identity function $i : (X, \Im) \to (X, \Im^*)$ defined by $i(x) = x$ to be continuous.

Section – B

Answer **all** questions. $16 \times 4 = 64$

3. (a) (i) If A, B, and E are subsets of a topological space $(X, \Im)$ then prove the following. If $A \subseteq B$ then $d(A) \subseteq d(B)$ and $d(A \cup B) = d(A) \cup d(B)$.

(ii) If F is a closed set in a topological space then CF is an open set.

OR

(b) (i) For any set E in a topological space, prove that $C(E) = E \cup d(E)$.

(ii) For a subset F of a topological space $(X, \mathfrak{I})$ if CF is an open set, then prove that F is a closed set.

4. (a) (i) If f is a continuous mapping of a topological space $(X, \mathfrak{I})$ into the topological space $(X^*, \mathfrak{I}^*)$, then prove that f maps every compact subsets X onto a compact subset of X^*.

(ii) Prove that if a connected set C has non-empty intersection with a set E and its complement in a topological space $(X, \mathfrak{I})$ then C has non-empty intersection with the boundary of E.

OR

(b) (i) If f is a one to one continuous mapping of $(X, \mathfrak{I})$ into $(X, \mathfrak{I}^*)$, then prove that f maps every dense in itself subset of X into dense in itself subset of X^*.

(ii) Prove that compactness is an absolute property.

5. (a) (i) Prove that a topological space is T_0 if and only if the closure of unequal points are unequal sets.

(ii) Prove that a topological space is T_1 if and only if every subset consisting of exactly one point is closed.

OR

(b) (i) Give example of a compact Housdroff space which does not satisfy the first axiom of countability.

(ii) Prove that every second axiom space is hereditarily separable.