2016
(January)

Time : 3 hours

Full Marks : 80

The figures in the right-hand margin indicate marks.

Answer from both the Sections as directed.

The symbols used have their usual meanings

(PARTIAL DIFFERENTIAL EQUATIONS AND ITS APPLICATIONS)

Section – A

1. Answer any **four** of the following : $4 \times 4 = 16$

   (a) Determine the general solution of

   $$u_{xx} - 3u_{xy} + 2u_{yy} = 0$$

   (b) State Cauchy problem for an infinite string. Hence, find its characteristic co-ordinates.

(c) Using variable separable method. obtain the soulution (s) of the Laplace's equation in polar co-ordinates.

(d) Show that the solution of the Dirichlet problem, if it exists, is unique.

(e) Find the Fourier cosine transform of $\frac{1}{1+x^2}$

(f) Find the inverse Laplace transform of $\frac{s+2}{s^2(s+1)(s-2)}$.

OR

2. Answer all questions :

$2\times8 = 16$

(a) Find the characteristic roots of the equation $z_{xx} + z_{yy} = 0$.

(b) What is the D'Alembert solution of the Cauchy problem, when $u(x, o) = \sin x$, $u_t(x, o) = \cos x$?

(c) State and prove the Lagrange's identity for operators.

(d) All the eigen values of a regular Sturm-Lioucville system are real. Explain it.

(e) State Dirichlet problem for a circle and a circular annulus.

(f) State Neumann problem for a rectangle and find its compatibility condition.

(g) Find the Fourier transform of

$$f(x) = \begin{cases} x^2, & |x| < x_o \\ o, & |x| > x_o \end{cases}$$

(h) State and explain the second shifting property for Laplace transform.

Section – B

Answer all questions :

3. (a) (i) Explain parabolic canonial form to solve a second order PDE . 6

(ii) Reduce the PDE

$$(n-1)^2 u_{xx} - y^{2n} u_{yy} - ny^{2n-1} u_y = 0$$

to canonical form and find its general solution. 10

OR

(b) (i) Deduce an expression to classify a second order PDE with variable coefficients. Hence, classify the PDE $x^2 u_{xx} + 2xy\, u_{xy} + y^2 u_{yy} = 0$. Reduce the given PDE to its concerned class. 10

(ii) Classify and reduce the following equation :

$y^2 u_{xx} - x^2 u_{yy} = 0$

to cannonical form. 6

4. (a) (i) State the Cauchy problem for Laplace equation. Hence, slove it. 8

(ii) Solve $u_{tt} - c^2 u_{xx} = 0$ subject to the conditions : $u(x, o) = \cos x$, $u_x(o, t) = 0 = u_x(x, t) = u_t(x, 0)$ for $t > 0$ and $0 \le x \le \bar{n}$. 8

OR

(b) (i) Find the solution of the initial boundary value problem

$u_{tt} - 4u_{xx} = 0, 0 < x < 1, t > 0$

subject to the conditions :

$u(x, 0) = 0 = u(0, t) = u(1, t), 0 \le x \le 1;$

$u_t(x, 0) = x(1 - x), 0 \le x \le 1.$ 8

(ii) State and prove the uniqueness theorem for one dimensional wave equation. 8

5. (a) (i) Find the Eigen values and Eigen functions of the regular Sturm-Liouville system :

$x^2 y^{11} + 3xy^1 + \lambda y = 0, \ 1 \le x \le e$

$y(1) = 0 = y(e).$ 8

(ii) Determine the solution of the problem :

$\nabla^2 u = 0, 1 < r < 2, 0 < \theta < \pi$

Subject to the conditions $u(2, \theta) =$

$\theta(\theta-\pi)$, $u(1,\theta) = 0 = u(r, 0) = u(r, \pi)$;

$0 \le \theta \le \pi$, $1 \le r \le 2$, 8

OR

(b) (i) Solve the Dirichlet problem :

$\nabla^2 u = 0$, $0 < x < 1$, $0 < y < 1$

subject to the conditions $u(x, 0) = x(x-1)$, $u(x, 1) = 0 = u(0, y) = u(1, y)$;

$0 \le x \le 1$, $0 \le y \le 1$. 8

(ii) Find the eigen values and eigen functions of the regular Sturm-Liouville system :

$$\frac{d}{dx}[(2+x)^2 y^1] + \lambda y = 0, \ -1 \le x \le 1,$$

$y(-1) = 0 = y(1)$ 8

6. (a) (i) Define Dirac delta function and find its Fourier transform. Using the formula solve the porblem :

$u_t = u_{xx} + \delta(x)\,\delta(t)$

subject to the conditions $u(x, o) = \delta(x)$, $\lim_{(x)\to\infty} u(x, t) = 0$ 10

(ii) Find the inverse Laplace transrorm of

$$\frac{1}{(s-a)^3} + \frac{\bar{e}^s}{(s-1)(s-2)}$$ 6

OR

(b) (i) Define convolution of two functions. State and prove the convolution theorem for Fourier transform. 6

(ii) Apply Laplace transform and solve :

$u_{tt} - c^2 u_{xx} = q(x, t)$, $x t \mathbb{R}$, $t > 0$

$u(x, 0) = f(x)$, $u_t(x, 0) = 9(x)$, $\forall x t \mathbb{R}$.

Express the solution in terms of D' Alembert's solution. 10

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