8. (a) Let r(t) be a positive continuous function and let m be a real number. Show that the equation $x'' + (m^2 + r(t))\, x = 0,\ t \geq 0$ is oscillatory.

(b) Show that the equation $x'' + \left(\frac{K}{t^2} - \frac{1}{t^n}\right)x = 0$, $n \geq 3,\ t > 1,\ K > \frac{1}{4}$ is oscillatory.

9. (a) Determine the nature of the critical points in the following systems :

$$x_1' = 4x_1 + x_2,\ x_2' = 3x_1 + 6x_2$$

(b) Determine the nature of the characteristic roots for the equation :

$$x''' + 3x'' + 2x' + x = 0$$

10. (a) The null solution of $x' = A(t)x$ is asymptotically stable if and only if $\| \phi(t) \| \to 0$ as $t \to \infty$.

(b) Show that the following system are asymptotically stable :

$$x_1' = -x_1^3 - x_1x_2^3$$

$$x_2' = x_1^4 - x_2^3$$

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# 2016

*Time : 4 hours*

*Full Marks : 100*

*The questions are of equal value.*

*Answer any five questions.*

## (ORDINARY DIFFERENTIAL EQUATIONS)

1. (a) Find the solution of the IVP :

$$x' = (1 + x^2)t,\ x(0) = 1$$

(b) Solve $xe^{x^2 + t^2}dx + t\,(e^{x^2 + t^2} + 1)dt = 0$.

2. (a) Find the particular solution by using the method of undetermined coefficients :

$$x'' + 25x = 2\sin 2t$$

(b) Find the general solution of the equation :

$$x'' - 9x' + 20x = t + e^{-t}$$

3. (a) Consider the system of equations :

$$x_1' = ax_1 + bx_2,\ x_2' = cx_1 + dx_2$$

Where a, b, c and d are constants, show that the above equation has a solution of the form

$\phi(t) = \alpha e^{rt}$, ($\alpha$ = constant), where r is a root of the equation $r^2 - r(a + d) + ad - bc = 0$.

(b) Find the determinant of fundamental matrix $\phi(t)$ which satisfies $\phi(0) = E$ for the system $x' = Ax$, where $A = \begin{bmatrix} -1 & 3 & 4 \\ 0 & 2 & 0 \\ 1 & 5 & -1 \end{bmatrix}$.

4. (a) Let f(t) be periodic with period w. Then a solution x(t) of $x' = Ax + f(t)$, $t \in (-\infty, \infty)$ is periodic of period w if and only if $x(0) = x(w)$.

(b) Find $e^{tA}$ when $A = \begin{bmatrix} -1 & 2 & 3 \\ 0 & -2 & 1 \\ 0 & 3 & 0 \end{bmatrix}$.

5. (a) Compute the first three succesive approximations for the solution of the following equation :

$$x' = \frac{x}{1 + x^2}, \; x(0) = 1$$

(b) Let f(t, x) be a continuous function in (t, x) on D. $x(t, t_0, x_0)$ denoted by x(t) is a solution of $x' = f(t, x)$, $x(t_0) = x_0 = (x_1(t_0), x_2(t_0), \ldots\ldots x_n(t_0))$ on same interval I contained in $|t - t_0| \le a$ if and only if x(t) is a solution of the internal equation $x(t) = x_0 + \int_{t_0}^{t} f(s, x(s))ds$, $t \in I$.

6. (a) Prove that the Euler's equation $x'' + \frac{K}{t^2}x = 0$ is oscillatory if $K > \frac{1}{4}$.

(b) Let $v, w \in c^1 \{(t_0, t_{0+h}), R\}$ be lower and upper solutions of $x' = f(t, x)$, $x(t_0) = x_0$, where $f \in C[D, R]$, where D is an open-connected set in $\mathbb{R}^2$ and $(t_0, x_0) \in D$ respectively. Suppose that for $x \ge y$, f satisfies the inequality $f(t, x) - f(t, y) \le L(x - y)$ ; where L is a positive constant. Then, $v(t_0) \le w(t_0)$ implies that $v(t) \le w(t)$, $t \in [t_0 + t_{0+h}]$.

7. (a) Show that the eigen values for the BVP $x'' + \lambda x = 0$, $x(0) = 0$ and $x(\pi) + x'(\pi) = 0$ satisfy the equation $\sqrt{\lambda} = -\tan(\pi\sqrt{\lambda})$. Prove that the corresponding eigen functions are $\sin(t\sqrt{\lambda n})$, where $\lambda_n$ is an eigen value.

(b) Solve the following BVP using the appropriate Green's function :

$x'' = et$, $x(0) = x(1) = 0$