(*b*) (*i*) Prove that the Wronskian of two solutions of the equation

$$a_0(x)y'' + a_1(x)y' + a_2(x)y = 0, \quad a_0(x) \neq 0$$

is either zero or never zero on (a, b).

(*ii*) State and prove the existence and uniqueness theorem for a Linear Differential Equation.

6. (*a*) (*i*) What do you mean by equations with deviating arguments ? Discuss its solution.

(*ii*) Write short note on Fundamental Matrix.

OR

(*b*) (*i*) Discuss the solution of linear system with periodic coefficient.

(*ii*) Discuss the formation of Linear Differential Equation of the first order and explain it by an example.

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2016

ORDINARY AND DIFFERENTIAL EQUATION - I

Time : Three Hours] [*Maximum Marks* : 80

The figures in the right hand margin indicate marks.
Answer from both the Sections as directed.

SECTION-A

1. Answer any **four** of the following : 4×4

(*a*) Show that $y = x + x \log x - 1$ is the unique solution of $xy'' - 1 = 0$, satisfying $y(1) = 0$ and $y'(1) = 2$.

(*b*) Solve $(D_2+1)\,y = \operatorname{cosec} x$

(*c*) Solve $xy'' + (2x-1)\,y' + (x-1)y = 0$

(*d*) Solve $(x^2y-2xy^2)\,dx - (x^3 - 2x^2y)dy = 0$

(*e*) Solve $(x^2D^2 - xD + 2)\,y = x \log x$

OR

2. Answer **all** questions : 2×8

(*a*) Define Homogeneous differential equation of first order and give an example.

(*b*) The integrating factor of the differential equation $(xy \sin xy + \cos xy)\, y\, dx + (xy \sin xy - \cos xy)\, x\, dy = 0$ is ....

(*c*) Find particular integral of the differential equation $(D^2 + a^2)\, y = \sin ax$

(*d*) If $y_1(x) = \sin 3x$ and $y_2(x) = \cos 3x$ are two solutions of $y''+9y = 0$, show that they are linearly independent.

(*e*) Find complementry function of the differential equation

$$xy^2 - (2x-1)y + (x-1)y = 0$$

(*f*) Solve $\frac{dy}{dx} - x\tan(y-x) = 1$.

(*g*) Find the equation to the curve for which Cartesian sub-normal is constant.

(*h*) Show that the equation $(1+x^2)\, y'' + 3xy' + y = 1+3x^2$ is exact.

SECTION-B

Answer **all** questions of the following : 16×4

3. (*a*) (*i*) Solve

$$(x-2y-3)dy + (x-y-2)dx = 0$$

(*ii*) Solve $(D^2 - 3D + 2)\, y = e^{2x} \sin x$

OR

(*b*) (*i*) Solve $(D^3 + 1)\, y = \cos 2x$

(*ii*) Solve $dx + y\, dy = m\, (dy - y\, dx)$

4. (*a*) (*i*) Solve $x^2y'' + xy' - y = 0$, given that $\left(x + \frac{1}{x}\right)$ is on integral.

(*ii*) Find the family of curves whose normal forms the angle $\frac{\pi}{4}$ with hyperbola $xy = c$

OR

(*b*) (*i*) Solve

$$y'' + 4y = \operatorname{cosec}^2 x$$

by the method of variation of parameter.

(*ii*) Solve $y'' - 2\tan x \,.\, y' + 5y = e^x \sec x$.

5. (*a*) (*i*) If $y = y_1(x)$ and $y = y_2(x)$ are two solutions of the equation $y'' + Py' + Qy = 0$, where P and Q are continuous functions of x, prove that

$$y_1 \frac{dy_2}{dx} - y_2 \frac{dy_1}{dx} = ce^{-\int P\, dx}$$, c is an arbitrary constant.

(*ii*) Solve $(x^2D^2 - 4xD + 6)y = x^4$

OR