2016

OPTIMIZATION TECHNIQUES - I

Time : Three Hours] [*Maximum Marks* : 80

The figures in the right hand margin indicate marks. Answer from both the Sections as directed.

SECTION-A

1. Answer any four of the following : 4×4

(*a*) Sketch Branch and Bound method in integer programming.

(*b*) Explain cutting plane method of solving an integer programming problem.

(*c*) Explain Kuhn-Tucker optimality conditions.

(*d*) What is Saddle Point ?

(*e*) What is Game Theory ? List out the assumptions made in the theory of games.

(*f*) Explain two person zero sum game.

OR

2. Answer **all** questions from the following : 2×8

(a) State the general form of integer programming problem.

(b) Distinguish between pure and mixed integer programs.

(c) Examine $z = 6x_1x_2$ for maxima and minima under the condition $2x_1+x_2 = 10$

(d) Find the dimensions of a rectangular parallelopiped with largest volume whose sides are parallel to the coordinate planes, to be inscribed in the ellipsoid

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

(e) Show that the (0,0) is saddle point for $f(x_1,x_2)=18x_1x_2+5x_2^2$

(f) What is Saddle Value Problem ?

(g) Define Fair Game.

(h) Explain pure and mixed strategies.

SECTION-B

Answer **all** questions : 16×4

3. (a) (i) Solve by the Gromery algorithm :

Minimize $Z = 20x_1+22x_2+18x_3$

subject to $4x_1+6x_2+x_3 \geq 54$

$4x_1+4x_2+6x_3 \geq 65$

x_1,x_2,x_3 each ≤ 7

All variables non-negative and integral.

(ii) Use cutting plane method to solve

Maximize $Z = x_1 + 2x_2$

Subject to $x_1 + x_2 \leq 7$

$2x_1 \leq 11,\ 2x_2 \leq 7$

$x_1, x_2 \geq 0$ and integer.

OR

(b) (i) Use Branch and Bound technique to solve

Maximize $Z = 3x_1+4x_2+5x_3$

subject to $x_1+x_2+x_3 \leq 10.5$

$2x_1+x_2+5x_3 \leq 15$

x_1,x_2 non-negative integer; $x_3 \geq 0$

(ii) Solve by additive algorithm :

Maximize $Z = 3x_1+2x_2-5x_3-2x_4+3x_5$

subject to $x_1+x_2+x_3+2x_4+x_5 \leq 4$

$7x_1+3x_3-4x_4+3x_5 \leq 8$

$11x_1-6x_2+3x_4-3x_5 \geq 3$

$x_j = 0$ or 1 for all j

4. (a) (i) State and prove Kuhn-Tucker necessary condition for optimization problems subject to inequality constraints.

(ii) Solve the following non-linear programming problem using the Lagrangian multipliers :

Optimize $Z = 4x_1^2+2x_2^2+3x_3^2-4x_1x_2$

subject to $x_1+x_2+x_3=15$

$2x_1-x_2+2x_3=20$

OR

(b) (i) Use the Kuhn-Tucker conditions to solve

Maximize $Z = 2x_1 - x_1^2 + x_2$

subject to $2x_1+3x_2 \leq 6$

$2x_1+x_2 \leq 4,\ x_1, x_2 \geq 0$

(ii) Solve the following by using Lagrangian multiplier :

Minimize $Z = x_1^2 + x_2^2 + x_3^2$

subject to $x_1+x_2+3x_3 = 2$

$5x_1+2x_2+x_3 = 5$

$x_1, x_2, x_3 \geq 0$

5. (a) (i) Define convex programming problem. What is the Lagrangian function associated with it ? Solve the problem.

Min $Z = -\log x_1 - \log x_2$ subject to

$x_1+x_2 \leq 2,\ x_1, x_2 \geq 0$

(ii) State and prove sufficient condition for non-negative saddle points.

OR

(b) (i) State and prove necessary conditions for saddle point correspondence.

(ii) Use the Kuhn-Tucker conditions to solve

Min. $f(x) = (x_1+1)^2 + (x_2+2)^2$

subject to $0 \leq x_1 \leq 2,\ 0 \leq x_2 \leq 1$

6. (*a*) (*i*) Solve the following (2×3) game graphically.

		B I	B II	B III
A	I	1	3	11
	II	8	5	2

(*ii*) In a game of matching coins with two players, suppose one player win Rs. 2 when there are two heads and wins nothing when there are two tails; and losses Re. 1 when there are one head and one tail. Determine the payoff matrix, the best strategies for each player and volume of game.

OR

(*b*) (*i*) Solve the following game using graphical method :

$$A \quad \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \overset{B}{\begin{array}{cc} B_1 & B_2 \\ \end{array}} \begin{bmatrix} 3 & -4 \\ 2 & 5 \\ -2 & 8 \end{bmatrix}$$

(*ii*) For any 2×2 two person zero sum game without saddle point, having payoff matrix for player *A* as

(*B*)		B_1	B_2
(*A*)	A_1	a_{11}	a_{12}
	A_2	a_{21}	a_{22}

Find optimal mixed strategies and value of game.

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