(b) (i) Given the initial value problem $u' = t^2 + u^2$, $u(0) = 0$, determine the first three non-zero terms in the Taylor series for $u(t)$ and hence obtain the value for $u(1)$. Determine t when the error in $u(t)$ obtained from the first two non-zero terms is to be less than 10^{-6} after rounding.

(ii) Solve the system of equation :

$$u' = 2u + v, \ u(0) = 1$$

$$v' = 3u + 4v, \ v(0) = 1$$

using Euler-Cauchy method with h=0.2 and evaluate u(0.4) and v(0.4).

2016
(January)

Time : 3 hours

Full Marks : 80

The figures in the right-hand margin indicate marks.

Answer from both the Sections as directed.

The symbols used have their usual meanings

(NUMERICAL ANALYSIS AND ITS APPLICATION)

Section – A

1. Answer any **four** of the following : 4×4 = 16

(a) If $f(x) = e^{ax}$, show that $\Delta^n f(x) = (e^{ah} - 1)^n e^{ax}$.

(b) Find the values of α, β and x_1, such that $\int_0^1 f(x)\,dx = \alpha f(0) + \beta f(x_1)$ is exact for polynomials of degree as high as possible.

(c) Show that interpolating polynomials are unique.

(d) Determine the step size h that can be used in the tabulation of $f(x) = \sin x$ in the interval [1, 3] so that the linear interpolation will be correct to four decimal places after rounding.

(e) Show that the error does not exceed 1/8 of second difference in the case of equispaced nodal points for linear interpolation.

(f) Derive Trapezoidal rule using methods based on undetermined coefficients.

OR

2. Answer **all** questions of the following :

$2 \times 8 = 16$

(a) Write down the formula for Bessel's interpolation.

(b) Define cubic splines.

(c) Show that $\Delta + \nabla = \Delta / \nabla - \nabla / \Delta$.

(d) Write down the error of approximation in the 2nd order derivative at any point x using methods based on interpolation.

(e) Show that $\mu = \left(1 + \frac{\delta^2}{4}\right)^{1/2}$.

(f) What is the truncation error in linear Lagrange interpolation method.

(g) Solve $\frac{du}{dt} = -2tu^2$, $u(0) = 1$. Find $u(0.4)$ using Euler's explicit method with $h = 0.2$.

(h) Write down the formula for fourth order Runge-Kutta method to solve the IVP.

Section – B

Answer **all** questions of the following : $16 \times 4 = 64$

3. (a) (i) Find the quadratic splines representing the function defined by the data

x	f(x)
–1	–4
0	1
1	0
2	5

Assume $f''(0) = M(0) = 0$. Interpolate at $x = 1.5$ and 2.5.

(ii) Derive the truncation error of the Lagrange quadratic interpolating polynomial $P_2(x)$, which coincides with the function $f(x)$ at x_0, x_1 and x_2.

OR

(b) (i) Construct the Hermite interpolation polynomial that fits the data :

x	f(x)	f'(x)
0	0	1.0000
0.5	0.4794	0.8776
1.0	0.8415	0.5403

Hence estimate the value of $f(0.75)$.

(ii) The following data represents the function $f(x) = e^x$.

x	f(x)
1	2.7183
1.5	4.4817
2.0	7.3891
2.5	12.1825

Estimate the value of $f(2.25)$ using Newton's forward difference interpolation. Compare with the exact value. Obtain the bound on the truncation error.

4. (a) (i) Obtain the least square polynomial approximation of degree one and two for $f(x) = x^3$ on the interval [0, 1] with $W(x) = 1$.

(ii) Using the following data obtain the Lagrange bivariate interpolating polynomial :

y/x	1	2	3
1	4	18	56
2	11	25	63
3	30	44	82

OR

(b) (i) Using the Chebyshev polynomials $T_n(x)$, obtain the least squares approximation of second degree for $3x^4 + 2x^3 + x + 2$ on $[-1, 1]$.

(ii) Obtain the rational approximation of the form $\frac{a_0 + a_1x}{1+b_1x+b_2x^2}$ to sin x.

5. (a) (i) Determine a, b and c such that the formula $\int_0^h f(x)\,dx = h\{af(0) + bf\left(\frac{h}{3}\right) + cf(h)\}$ is exact for polynomials of as high order as possible and determine the truncation error.

(ii) Derive the formulas for the first and second derivatives of y = f(x) using forward difference approximations :

x	f(x)
$\pi/4$	.7071
$\pi/3$	.8660
$5\pi/12$	.9656
$\pi/2$	1

Estimate $f^1\left(\frac{\pi}{4}\right)$ with $h = \frac{\pi}{12}$. Obtain the bounds on the truncation error and compare with the exact solutions.

OR

(b) (i) Evaluate:

$$\int_{\pi/4}^{\pi/2} \frac{\cos x \ln(\sin x)}{\sin^2 x+1}\,dx$$

Correct to three decimal places using Romberg integration.

(ii) Evaluate the double integral

$$\int_0^1\int_1^2 \frac{2xy}{(1+x^2)(1+y^2)}\,dydx$$

using the trapezoidal rule with h = k = 0.25.

6. (a) (i) Solve the differential equation :

$$u_{j+1} - 2(\sin x)\, u_j + u_{j-1} = 0$$

when $u_0 = 0$ and $u_1 = \cos x$.

(ii) Solve the I. V. P. $u' = -2tu^2$, u (0) = 1 estimate u(0.5) using the third order Adams-Bashforth method on [0, 1] with h = 0.2.

OR