(b) If A and B are regular matrices and A is μ_n-stronger than B, then show that A and B are ρ_n consistent for some $\rho_n \uparrow \infty$ $(\rho_n \le \mu_n)$.

8. (a) Show that if $\{s_n\}$ belongs to the unit ball, A be a regular matrix and A-lim $s_n = N(A) > 1$, then $\limsup_{n \to \infty} s_n = 1$ and $\liminf_{n \to \infty} s_n = -1$.

(b) Prove that if A and B are regular matrices and $A \supseteq B$ then $\|A\| \ge \|B\|$.

9. (a) Let $0 < p < 1$ Show that $A \in (w_p, c)$, if and only if :

(i) $a_{xk} \to a_k$, $n \to \infty$, k be fixed

(ii) $M = \sup_n \sum_{r=0}^{\infty} 2^{r/p} A_r^1(n) < \infty$

(b) Let $1 < p < \infty$ and suppose that $A \in (l_\infty, l_\infty) \cap (l_1, l_1)$ show that $A \in (l_p, l_p)$.

10. (a) Prove that every normal co-null matrix is stronger than convergence.

(b) Let $p = (p_k)$ be bounded and A non-negative then $[A, p]_0$, $[A, p]$, and $[A, p]_\infty$ are linear spaces.

2016

Time : 4 hours

Full Marks : 100

The questions are of equal value.

*Answer any **five** questions.*

Symbols used have their usual meanings.

(MATRIX TRANSFORMATION IN SEQUENCE SPACES)

1. (a) Define Regular Transformation. Show that Cesaro method is regular.

(b) Show that the matrix A transforms all bounded sequences into bounded sequences if and only if it transforms all sequences which converges to 0 into bounded sequence.

2. (a) Prove that to every bounded sequence σ there corresponds a matrix A of the class L such that the A-limit of σ exists.

(b) Show that the necessary and sufficient conditions for the matrix $A = (a_{mn})$ limits all bounded sequences to 0 is that the series $\sum_{n=1}^{\infty} |a_{mn}|$ cgs for every m and $\sum_{n=1}^{\infty} |a_{mn}| \to 0$ as $m \to \infty$.

3. (a) Define regular Norlund mean. If (N, p_n) and (N, q_n) are two regular Norlund means, then show that $(N, q_n) \supseteq (N, p_n)$ if and only if there is an M such that for every n, $|K_1|p_n + |K_2| p_{n-1} + \ldots\ldots\ldots + |K_n|p_1 \leq MQ_n$, and $\lim_{n \to \infty} \frac{K_n}{Q_n} = 0$.

(b) For every positive integer K, show that the Cesaro matrices (C, K) satisfy the following inclusion relation $(C, K + 1) \supseteq (C, K)$.

4. (a) Define moment sequence and Housdorff matrix. Show that the method (C, K) is a Housdorff method corresponding to the function $\phi(x) = K\int_0^x (1-t)^{K-1} dt$.

(b) Define Abel's Method. Show that it is regular.

5. (a) Prove that the functional $P(s_n)$ is sublinear, where $\{s_n\}$ is a bounded sequence and the functional given by $P(s_n) = \inf_{n_1, n_2 \ldots n_k} \limsup_{j \to \infty} \frac{1}{K} \sum_{p=1}^{k} s_{n_p + j}$, K is a positive integer and $x_1, x_2, \ldots\ldots, n_K$ are abitrary subsets of the integers.

(b) Prove that a matrix $A = (a_{m,n})$ is strongly regular if and only if it is translative.

6. (a) Let both $A = (a_{m,n})$ and $B = (b_{m,n})$ be regular and triangular matrices and let A be a perfect matrix. Show that if $O(A) \subseteq O(B)$ then B is a stronger than A.

(b) Define G_δ set. Show that if S is the complement of an every dense set of type G_δ, then S is of the first category.

7. (a) If A and B are regular matrices such that $A \supset B$, then show that there exists a regular matrix C such that $A \supset C \supset B$.