2016

Time : 3 hours

Full Marks : 80

The figures in the right-hand margin indicate marks.

Answer from both the Sections as directed.

(MATHEMATICAL STATISTICS – I)

Section – A

1. Answer any **four** of the following : $4 \times 4 = 16$

   (a) Find the chance of throwing a five or a six at least once in four throws of an unbiased dice.

   (b) If A and B are independent events then prove that (i) A and $\bar{B}$ are independent (ii) $\bar{A}$ and $\bar{B}$ are independent.

   (c) If an unbiased coin is tossed n times, find the mathematical expectation of the number of heads in all n tosses.

(d) State and prove Cauchy-Schwarz inequality.

(e) If X is a Poisson random variable such that $P(X = 2) = q\,p\,(X = 4) + 90p\,(X = 6)$, find the coefficient of skewness.

OR

2. Answer **all** questions from the following : $2 \times 8 = 16$

(a) Two dice are thrown. What is the probability of getting total ≤ 11 ?

(b) Give an example of geometric probability distribution.

(c) What is the basic difference in discrete and probability distribution ?

(d) Two dice are thrown. Find the probability that the sum is neither 6 nor 10.

(e) If a random variable x has a cumulative distribution function

$$F(x) = \begin{cases} 0 & \text{for} \quad x \leq 0 \\ c\left(1 - e^{-x}\right) & , \quad x > 0 \end{cases}$$; what is $P(1 \leq x \leq 2)$?

(f) Define coefficient of variation and co-efficient of skewness.

(g) Find the mean of uniform density function.

(h) If the probability of sucess is 0.09, how many trials are needed to have a probability of at least one success as $\frac{1}{3}$ or more ?

Section – B

Answer **all** questions : $16 \times 4 = 64$

3. (a) State and established general addition law in probability.

OR

(b) State and establish Baye's theorem.

4. (a) A continuous random variable X has the density function f(x) given by :

(i) $f(x) = \dfrac{c}{x^2 + 1}, \; -\infty < x < \infty$

(ii) $f(x) = \begin{cases} 0 & \text{if} \quad x \leq 0 \\ cxe^{-x} & \text{if} \quad x > 0 \end{cases}$

(iii) $f(x) = \begin{cases} cx^2 & \text{if } 0 < x < 3 \\ 0, & \text{otherwise} \end{cases}$

Find the distribution function after determining the value of C.

OR

(b) X is a continuous random variable with p. d. f. given by :

$$f(x) = \begin{cases} ax & \text{if } 0 \le x \le 1 \\ a & \text{if } 1 \le x \le 2 \\ 3a - ax & \text{if } 2 \le x \le 3 \\ 0 & \text{otherwise} \end{cases}$$

Find the value of a cumulative distibution function f(x) and hence obtain $\rho(x \le 1.5)$.

5. (a) Find the characteristic function, mean and variance of the random variable X with $f_X(n) = \frac{a^{n+1}x^n}{n!}e^{-ax}$, $x \ge 0$ and $f_X(n) = 0$, otherwise.

OR

(b) (i) Define Cauchy-Schwarz inequality and establish it.

(ii) Define and establish Markov inequality.

6. (a) Show that Poisson distribution is a limiting form of the binomial distribution.

OR

(b) Find the coefficient of skewness and kurtosis of a Poisson distribution with mean m.