4. (a) Define integrability of a measurable function. State and prove Lebesgue convergence theorem.

OR

(b) State and prove bounded convergence theorem.

5. (a) If f is absolutely continuous on [a, b] and $f'(x) = 0$ a. e., then prove that f is constant.

OR

(b) State and prove Vitali's Lemma.

6. (a) State and prove Minkowski Inequality.

OR

(b) State and prove Riesz-Fischer Theorem.

2016

Time : 3 hours

Full Marks : 80

The figures in the right-hand margin indicate marks.

Answer from both the Group as directed.

(ABSTRACT MEASURE)

Group – A

1. Answer any **four** of the following : 4×4 = 16

(a) If f is a measurable function and f = g a. e., then prove that g is measurable.

(b) Show that if $f(x) = \begin{cases} 0 & x \text{ irrational} \\ 1 & x \text{ rational} \end{cases}$, then

$$R\int_a^{\bar{b}} f(x)dx = b - a \text{ and } R\int_a^b f(x)\,dx = 0.$$

(c) Show that if f is intergonal over E, then so is $|f|$ and $\left|\int_E f\right| \le \int_E |f|$.

(d) Let f defined by $f(x) = \begin{cases} 0, & x = 0 \\ x^2 \sin\frac{1}{x}, & x \neq 0 \end{cases}$

Test whether f is of bounded variation on [–1, 1].

(e) Prove that every convergent sequence is a Cauchy sequence.

OR

2. Answer **all** questions from the following :

$2 \times 8 = 16$

(a) Let f be an extended real valued function whose domain is measurable. If for each real number α, the sets $\{x : f(x) \ge \alpha\}$ and $\{x : f(x) \le \alpha\}$ are measurable, show that for every extended real number α, the set $\{x : f(x) = \alpha\}$ is measurable.

(b) Give example of a non-measurable set.

(c) Define functions of bounded variation.

(d) Define a convex function. Give its geometric interpretation.

(e) Prove that $\|f + g\|_1 \le \|f\|_1 + \|g\|_1$.

(f) Define Δ-approximant to an integrable function on a closed interval.

(g) Define absolute continuity of a real valued function on a closed interval.

(h) If $g(x) = f(-x)$ then show that :

$D^+g(x) = -D_-f(x)$.

Group – B

Answer **all** questions of the following : $16 \times 4 = 64$

3 (a) Show that the outer measure of an interval is its length.

OR

(b) Prove that the interval (a, ∞) is measurable.