2017

Time : 3 hours

Full Marks : 80

The figures in the right-hand margin indicate marks.

Answer from both the Sections as directed.

(Mathematical Statistics-I)

GROUP—A

1. Answer any *four* of the following questions : 4×4

(*a*) A bag contains 50 tickets numbered 1,2,3,...,50 of which five are drawn at random and arranged in ascending order of magnitude. What is the probability that the middle one is 30 ?

(*b*) A discrete random variable X has following probability distribution :

x	0	1	2	3	4	5	6	7
$p(x)$	0	a	$2a$	$2a$	$3a$	a^2	$2a^2$	$7a^2 + a$

Find $P(0 < X < 4)$.

(c) If probability density function of a continuous random variable is given by

$$f_x(x) = \begin{cases} c(3+2x), & 0 < x < 2 \\ 0 & , \text{ otherwise} \end{cases}$$

find the distribution function free from c.

(d) A coin is tossed until a head appears. Find the expectation of the number of tosses required.

(e) X be a random variable having Poisson distribution such that $P(X=1)=0.3$ and $P(X=2)=0.2$. Find $E(X)^2$.

Or

2. Answer *all* questions of the following : 2×8

(a) Four cards are drawn from a well-shuffled pack of cards. What is the probability that there is one from each suit ?

(b) A , B , C be events associated with an experiment $\{A, B, C\}$. If

$$P(B) = \frac{3}{2} P(A), \ \ P(C) = \frac{1}{3} P(B),$$

find $P(A)$.

(c) If the p.d.f. of a random variable X is given as

$$f(x) = \begin{cases} \dfrac{c}{x^3}, & 1 < x < \infty \\ 0 & , \text{ otherwise} \end{cases},$$

find c.

(d) If p.d.f. of a continuous random variable X is given by

$$f(x) = \begin{cases} \dfrac{1}{2}x, & \text{if } 0 < x < 2 \\ 0 & , \text{ otherwise} \end{cases},$$

find $E(X^2)$.

(e) Find the moment generating function of the random variable X which assumes values -1 and $+1$ with probabilities $\frac{1}{2}$ and $\frac{1}{2}$ each.

(f) Determine the mode of the binomial distribution whose mean is 4 and variance is 3.

(g) X be a Poisson variate. It is known that the frequencies of X taking values 3 and 4 are equal. Obtain $P(X=0)$.

(h) If a hunter is shooting a target, what is the probability that 5th fire is the 2nd hit, given the probability of hitting the target at any trial is 0.20 ?

GROUP—B

Answer *all* questions. 16×4

3. (*a*) A player tosses a coin and is to score one point for every head turned up and two for every tail. He is to play on until his score reaches or passes n. If p_n is the probability for attaining exactly n, show that

$$p_n = \frac{1}{2}(p_{n-1} + p_{n-2}) \text{ and hence find } p_n$$

Or

(*b*) State Bay's theorem. An urn contains 5 white and 5 black balls; 4 balls are drawn from this urn and put into another urn. Then from the 2nd urn a ball is drawn and found to be white. Find the probability of drawing another white ball at the next draw when the first white ball is not replaced.

4. (*a*) A continuous random variable X has density functions given by

(*i*) $$f(x) = \begin{cases} \frac{c}{x^3}, & \text{if } 1 < x < \infty \\ 0, & \text{otherwise} \end{cases}$$

(*ii*) $$f(x) = \begin{cases} cx & \text{if } 0 \le x < 2 \\ 2c & \text{if } 2 \le x < 4 \\ 6c - cx & \text{if } 4 \le x < 6 \end{cases}$$

In each case find c, distribution function $F(x)$ and $P(3 \le x < 5)$.

Or

(*b*) The waiting time in hours, between two successive speeders spotted by a radar unit is a continuous randon variable with cumulative distribution :

$$F_X(x) = \begin{cases} 0, & x \le 0 \\ 1 - e^{-8x}, & x > 0 \end{cases}$$

Find the probability of waiting less than 12 minutes between successive speeders

(*i*) using the cumulative distribution of X,

(*ii*) using probability density function of X.

5. (*a*) X is a random variable with density function

$$f(x) = \begin{cases} Ae^{x}, & 0 < x < \infty \\ 0, & \text{otherwise} \end{cases}$$

Find the value of A, mean of X, variance of X, third moment about the mean, kurtosis and r th moment about origin.

Or

(*b*) The density function of a random variable X is given by $f(x) = Kx(2-x), \ 0 \le x < 2$. Find K, mean, variance (X), β_1, β_2, rth moment, mode and median of the distribution.

6\. (*a*) Define Geometric distribution. Find mean, variance and moment generating function for this distribution.

Or

(*b*) Define Poisson distribution. Find its mean, variance, moment generating function.