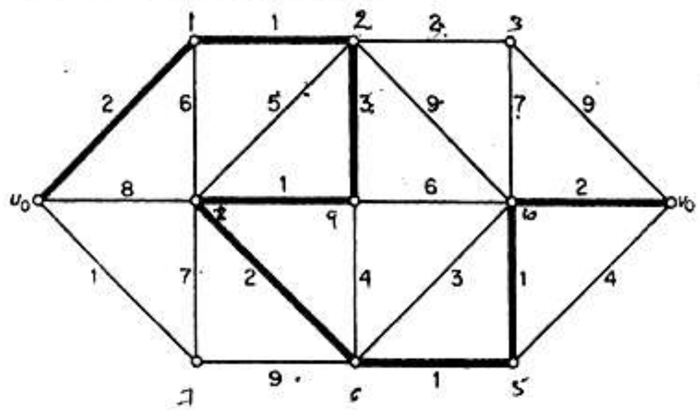
(b) Find the shortest paths from u_0 to all other vertices in the following weighted graph



6. (a) (i) Prove that a connected graph is Eulerian if and only if every vertex has even degree.

(ii) Prove that a connected planner graph with n vertices and e edges has $e + n - 2$ regions

OR

(b) State and prove Kuratowski theorem.

M.A./M.Sc.-Math.-IVS-(CC-401)

2017

Time : 3 hours
Full Marks : 80

The figures in the right hand margin indicate marks.
Answer from both the Sections as directed.

(GRAPH THEORY)

SECTION – A

1. *Answer any **four** of the following :* ***(4x4=16)***

(a) Prove that every graph G with at least one edge has a sub-graph H with $\delta(H) > \varepsilon(H) \geq \varepsilon(G)$.

(b) Show that an infinite graph with finite number of vertices (i.e. a graph with finite number of vertices and infinite number of edges) will have at least one pair of vertices joined by an infinite of parallel edges.

(c) Show that Hamiltonian path is spanning tree.

(d) Prove that any given edge of connected graph G is a branch of some spanning tree of G.

(e) The spherical embedding of every planner 3-connected graph is unique.

(f) Prove that in a complete graph with n vertices, there are $(n-1)/2$ edge-disjpint Hamiltonian circuit, if n is odd number ≥ 3.

(Turn over)

OR

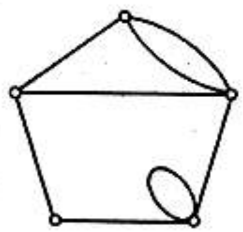
2. *Answer **all** questions :* *(2x8=16)*

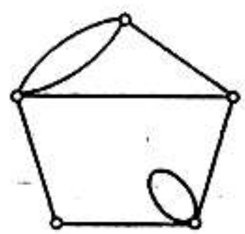
(a) Prove that the number of vertices of odd degree in a graph is always even.

(b) Prove that a connected graph with n vertices is a tree if and only if it has $n-1$ edges.

(c) What is the number of edge in a K^n?

(d) Show that the following graphs are not isomorphic.





(e) Show that if G is disconnected then G^c is connected.

(f) Describe the Dijkstra's algorithm

(g) In any simple, connected planner graph with f regions, n vertices, and e edges $(e > 2)$ prove that $e \leq 3n - 6$ and $e \geq \frac{3}{2}f$

(h) State Dirac theorem for Hamiltonian cycles.

(Turn over)

SECTION – B

*Answer **all** questions* *(16x4=64)*

3. (a) Prove that every graph of average degree at least $4k$ has a $k-$ connected sub graph.

OR

(b) Let G be a graph containing a cycle C, and assume that G contains a path of length k between two vertices of C. Show that G contains a cycle of length at least $\sqrt{k}$. Is this the best possible?

4. (a) Prove that the number of labeled trees with n vertices $(n \geq 2)$ is n^{n-2}

OR

(b) (i) Show that any tree T has at least $\Delta(T)$ leaves.

(ii) Show that every automorphism of a tree fixes a vertex or a edge.

5. (a) Prove that the cycle space of a 3-connected graph is generated by its non separating induced cycles.

OR

(Turn over)