MA/M.Sc–Math-IIS- (203)

2017

Time : 3 hours

Full Marks : 80

The figures in the right-hand margin indicate marks.

Answer from both the Sections as directed.

(Algebra-II)

SECTION—A

1. Answer any *four* of the following questions : 4×4

(*a*) Show that $A(W)$ is a subspace of $\hat{V}$

(*b*) For $A, B \in F_n$ then prove that

$$tr\,(AB) = tr\,(BA).$$

(*c*) $T \in A(V)$ is unitary iff $TT^* = 1$.

(*d*) Consider $T : F^3 \to F^3$ such that $T(e_1) = e_2$,

$$T(e_2) = e_3,\ T(e_3) = e_4 \text{ and } mT = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

Find the minimal polynomial for T.

(Turn Over)

(e) If G is solvable group, then prove that $G^{(K)} = (e)$, for some integer K.

(f) If $u, v \in V$ then prove that

$$|\langle u, v \rangle| \le \|u\| \ \|v\|.$$

Or

2. Answer *all* questions from the following : 2×8

(a) Define a nilpotent operator and a nilpotent matrix.

(b) Write down the Jordan blocks of order 1, 2 and 3 belonging to the eigenvalue $\lambda = 3$.

(c) Suppose A is a 5×5 square matrix with minimal polynomial $m(t) = (t-2)^2$. Find Jordan canonical form.

(d) Is $A = \begin{pmatrix} 2 & 5 \\ 3 & -1 \end{pmatrix}$ is normal ?

(e) Define solvability by radicals over a field F.

(f) Define an annihilator of W.

(g) Define A^* for the matrix

$$A = \begin{pmatrix} 2+3i & 2-i \\ 1-2i & 2-2i \\ 3+4i & 2+i \end{pmatrix}$$

(h) Find the basis $\{f_1, f_2, f_3\}$ which is dual to the usual basis $\{e_1, e_2, e_3\}$ of $\mathbb{R}^3$.

SECTION—B

Answer *all* questions. 16×4

3. (a) If V is finite-dimensional and W is a subspace of V, then $\hat{W}$ is isomorphic to $\bar{V} / A(W)$ and $\dim A(W) = \dim V - \dim W$.

Or

(b) If K is a finite extension of F, the $G(K, F)$ is a finite group and its order, $o(G(K, F))$ satisfies $o(G(K, F)) \le [K : F]$.

4. (a) Let $V = F^{(3)}$ and suppose that $\begin{pmatrix} 1 & 1 & 2 \\ -1 & 2 & 1 \\ 0 & 1 & 3 \end{pmatrix}$

in the matrix of $T \in A(V)$ in the basis $v_1 = (1,0,0)$, $v_2 = (0,1,0)$, $v_3 = (0,0,1)$. Find the matrix of T in the basis $u_1 = (1,1,1)$, $u_2 = (0,1,1)$, $u_3 = (0,0,1)$.

Or

(*b*) If V is finite dimensional over F, then $T \in A(V)$ is regular iff T maps V onto V.

5. (*a*) If V is n-dimensional over F and if $T \in A(V)$ has all its characteristic roots in F, then T satisfies a polynomial of degree n over F.

Or

(*b*) For each $i = 1, 2, \dots K$, $V_i \neq (0)$ and $V = V_1 \oplus V_2 \oplus \dots \oplus V_K$, prove that the minimal polynomial of T_i is $q_i(x)^{l_i}$.

6. (*a*) The linear transformation T on V is unitary if and only if it takes an orthonormal basis of V into an orthonormal basis of V.

Or

(*b*) For $A, B \in F_n$, then prove that

(*i*) $\det(AB) = (\det A)(\det B)$.

(*ii*) If A is invertible then for all B, $\det(ABA^{-1}) = \det(B)$.