

Or

(b) Show that $\Gamma\left(\frac{1}{6}\right) = 2^{-\frac{1}{2}} \left(\frac{3}{\pi}\right)^{\frac{1}{2}} \Gamma\left(\frac{1}{3}\right)^2$

5. (a) Prove that an entire function of fractional order assumes every finite value infinitely many times.

Or

(b) Prove that the function $\tau(s) = \frac{1}{2}s(1-s)$

$\pi^{-\frac{s}{2}} \Gamma\left(\frac{s}{2}\right) \tau(s)$ is entire and satisfies $\tau(s) = \tau(1-s)$.

6. (a) Prove that an elliptic function without poles is a constant.

Or

(b) Prove that $p'(z) = -\sigma(2z) |\sigma(z)|^4$.

MA/M.Sc-Math-IIS- (204)

2017

Time : 3 hours

Full Marks : 80

The figures in the right-hand margin indicate marks.

Answer from both the Sections as directed.

(Advanced Complex Analysis)

SECTION—A

1. Answer any four of the following questions : 4 × 4

(a) Evaluate $\int_0^{\infty} y^2 e^{-2y^2} dy$

- (b) Investigate the convergence of the infinite

product : $\prod_{k=1}^{\infty} \left(1 + \frac{1}{k}\right)$

- (c) Expand $f(z) = \sin z$ in a Taylor series about

$z = \frac{\pi}{4}$.

(d) Evaluate $\int_C \bar{z} dz$ from $z = 0$ to $z = 4 + 2i$ along the curve C given by $z = t^2 + it$.

(e) Find the fixed points of the transformation

$$w = \frac{2z-5}{z+4}.$$

(f) Find the value of $\oint_C \frac{\sin 6z}{z - \frac{\pi}{6}} dz$, when

$$C : |z| = 1.$$

Or

2 × 8

2. Answer all questions from the following :

(a) Determine the singularity of the function

$$\frac{z}{e^z - 1}.$$

(b) Find the value of

$$\left(1 - \frac{1}{2^2}\right) \left(1 - \frac{1}{3^2}\right) \left(1 - \frac{1}{4^2}\right) \dots$$

(c) Define simple periodic functions and double periodic functions.

(d) What is the value of $\Gamma\left(-\frac{1}{3}\right)$.

(e) Define Riemann zeta function.

(f) Find the poles and residues of $\frac{1}{(z^2-1)^2}$.

(g) Is $\sin z$ is bounded? Justify your answer.

(h) State fundamental theorem of algebra.

SECTION—B

Answer all questions.

16 × 4

3. (a) Evaluate $\int_0^{\infty} \frac{dx}{x^6+1}$.

Or

(b) If a function $f(z)$ is analytic on R , then show

$$\text{that } \int_{\partial R} f(z) dz = 0.$$

4. (a) Develop $\log\left(\frac{\sin z}{z}\right)$ in powers of z up to the term z^6 .