**MA/M.Sc–Math-IIS- (202)**

**2017**

*Time* : 3 hours

*Full Marks* : 80

*The figures in the right-hand margin indicate marks.*

Answer from both the Groups as directed.

**( Advanced Calculus )**

GROUP—A

1. Answer any *four* of the following questions : 4 × 4

(*a*) Using sufficient condition of differentiability show that

$$f(x,y)=\begin{cases} xy\dfrac{x^2-y^2}{x^2+y^2}, & x^2+y^2\neq 0 \\ 0 \quad\quad\quad\quad , & x=y=0 \end{cases}$$

is differentiable at (0, 0).

(*b*) Show that $z=f\left(x^2y\right)$, where $f$ is differentiable, satisfies $x\left(\dfrac{\partial z}{\partial x}\right)=2y\left(\dfrac{\partial z}{\partial y}\right)$.

(c) If $V = F(x, y)$ and $x = e^u \cos t$, $y = e^u \sin t$,

show that $\dfrac{\partial^2 v}{\partial u^2} + \dfrac{\partial^2 v}{\partial t^2} = e^{2u}\left(\dfrac{\partial^2 v}{\partial x^2} + \dfrac{\partial^2 v}{\partial y^2}\right)$.

(d) Find the stationary points of the function $xy^2z^3$ subject to the conditions $x + y + z = 6$, $x, y, z > 0$.

(e) Compute $\int_\Gamma \dfrac{dx}{x+y}$ where $\Gamma$ is the curve

$x = at^2$, $y = 2at$, $0 \le t \le 2$.

*Or*

2. Answer all questions of the following : 2 × 8

(a) If $f(x, y) = \sqrt{|xy|}$, find $f_x(0, 0)$.

(b) Write down the Taylor's theorem for expansion of $f(x, y)$ about $(a, b)$ with remainder term.

(c) $u = \phi(y) + \psi(z)$ where $y = x + at$, $z = x - at$,

show that $\dfrac{\partial^2 u}{\partial y \partial z} = 0$.



(d) If $u = F(x, y, z)$ and $z = f(x, y)$, find $\dfrac{\partial^2 u}{\partial y^2}$ in terms of derivatives of $F$ and $f$.

(e) Examine the equation $y^2 + yx^3 + x^2 = 0$ for the existence of unique implicit function near (0, 0).

(f) Show that for $f(x, y) = y^2 - yx^2 - 2x^5 = 0$,

$y = \dfrac{x^2}{z}\left(1 - \sqrt{1+8x}\right)$, $x > -\dfrac{1}{8}$ is the unique solution in the nbd of (1, – 1).

(g) Evaluate $\iint_R \dfrac{x-y}{x+y}\, dx\, dy$, $R = [0, 1 ; 0, 1]$

(h) Change the order of integration :

$$\int_{-1}^{1} dx \int_{0}^{\sqrt{1-x^2}} f\, dy.$$

GROUP—B 16 × 4

3. (a) Find first four terms of the Maclaurin expansion of $e^{ax} \cos by$. Also find the remainder after four terms.

*Or*

(*b*) Show that for the function

$$f(x,y)=\begin{cases}\dfrac{x^2y^2}{x^2+y^2}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$$

$f_{xy}(0,0)=f_{yx}(0,0)$, even though the conditions of Schwarz's theorem and also of Young's theorem are not satisfied.

4. (*a*) If $F$ is a function of $x$ and $y$ and that $x=e^u+e^{-v}$, $y=e^v+e^{-u}$, prove that

$$\frac{\partial^2 F}{\partial u^2}-2\frac{\partial^2 F}{\partial u\partial v}+\frac{\partial^2 F}{\partial v^2}=x^2\frac{\partial^2 F}{\partial x^2}-2xy\frac{\partial^2 F}{\partial x\partial y}+y^2\frac{\partial^2 F}{\partial y^2}+x\frac{\partial F}{\partial x}+y\frac{\partial F}{\partial y}$$

2.

*Or*

(*b*) If $u=y^2+z^{-2}$, $v=z^2+x^{-2}$, $w=x^2+y^{-2}$ and if $V$ is a function of $x, y, z$, prove that

$$x\frac{\partial V}{\partial x}+y\frac{\partial V}{\partial y}+z\frac{\partial V}{\partial z}+2\left(u\frac{\partial V}{\partial u}+v\frac{\partial V}{\partial v}+w\frac{\partial V}{\partial w}\right)=4\left(y^2\frac{\partial V}{\partial u}+z^2\frac{\partial V}{\partial v}+x^2\frac{\partial V}{\partial w}\right)$$



5. (*a*) If $u, v, w$ are roots of the equation $(\lambda-x)^3+(\lambda-y)^3+(\lambda-z)^3=0$ in $\lambda$, prove that $$\frac{\partial(u,v,w)}{\partial(x,y,z)}=-2\frac{(y-z)(z-x)(x-y)}{(v-w)(w-u)(u-v)}.$$

*Or*

(*b*) Find the shortest distance from origin to the parabola $x^2+8xy+7y^2=225$, $z=0$.

6. (*a*) Prove that

$$\beta(m,n)=\frac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)},\quad m>0,\ n>0.$$

*Or*

(*b*) With the help of Green's formula compute the difference between the integrals

$$I_1=\int_{ACB}\{(x+y)^2dx-(x-y)^2dy\}$$ and

$$I_2=\int_{ADB}\{(x+y)^2dx-(x-y)^2dy\}$$

where $ACB$ and $ADB$ are respectively the straight line $y=x$ and the parabolic arc $y=x^2$, joining the points $A\,(0,0)$ and $B\,(1,1)$.