5. (*a*) (*i*) State and prove Tychonoff Lemma.

(*ii*) Prove that X is normal if and only if for each pair of disjoint closed subsets F_1 and F_2 of X there exits, a continuous function $g : X \to [a,b]$, $(a < b)$, such that $g(F_1) = \{a\}$ and $g(F_2) = \{b\}$.

OR

(*b*) (*i*) Show that every Lindelöf space is normal.

(*ii*) State and prove Tietze's extension theorem.

6. (*a*) (*i*) State and prove Urysohn's Metrization theorem.

(*ii*) If (X, J) be a regular space whose topology J has a σ-locally finite base B. Then show that X is a normal space.

OR

(*b*) (*i*) If X be a T_3 space whose topology J has a σ-locally finite base, then show that X is metrizable.

(*ii*) Show that metrizability is hereditary and topological invariant.

———

January, 2017

TOPOLOGY

Time : Three Hours] [*Maximum Marks* : 80

Answer from both the Sections as directed. The figures in the right-hand margin indicate marks.

SECTION-A

1. Answer any **four** of the following : 4×4

(*a*) If A be a subset of space X, show that $A \cup D(A)$ is closed subset of X.

(*b*) Show that if X is a countably compact, then it has Bolzano-Weierstrass property.

(*c*) Show that connectedness is topological property.

(*d*) Define Fort space and show that Fort space is normal.

(*e*) Show that every subspace of a completely regular space is also completely regular.

(*f*) Show that for every open covering of a metric space (X, d) there is a σ-discrete open cover which refines it.

OR

2. Answer **all** the questions from the following : 2×8

(a) Define bases for a topology with example.

(b) What is equivalent bases ? Write condition for equivalence of bases.

(c) Define compact space with example.

(d) If $f : X \to Y$ be continuous and onto and A is a dense subset of X, show that $f(A)$ is a dense subset of Y.

(e) Define completely normal space.

(f) Define T_2 space with example.

(g) Define locally finite family with example.

(h) If X and Y are separable spaces, then $X \times Y$ is a separable space, prove it.

SECTION-B

Answer **all** the questions : 16×4

3. (a) (i) If A be a subset of X, then show that the interior of A is the complement of the closure of the complement of A and the closure of A is the complement of the interior of the complement of A.

(ii) If A and B be subsets of a topological space X and A is open in X, then show that $A \cap \overline{B} \subset \overline{A \cap B}$.

OR

(b) (i) State and prove the criteria for bases for a topology.

(ii) Give an example of two subsets A and B on the real line $\mathbb{R}$ such that the four sets $\overline{A} \cap B, A \cap \overline{B}, \overline{A \cap B}$ and $\overline{A} \cap \overline{B}$ are all distinct.

4. (a) (i) Show that a topological space $(X, \mathcal{J})$ is compact if every family of sub-basic closed sets with finite intersection property has non-empty intersection.

(ii) If $h : (X, \rho_1) \to (Y, \rho_2)$ be a bijection. Then show that h is a homomorphism if and only if h is open and continuous or h is closed and continuous.

OR

(b) (i) If $f : X \to Y$ be continuous and onto, show that if X is a Lindelöf space then Y is also Lindelöf space.

(ii) In a topological space X show that following statements are equivalent :

(S_1) X is disconnected

(S_2) $X = A \cup B$ where A and B are two non-empty disjoint closed sets

(S_3) X has a non-empty proper closen subset