6. (a) Solve the initial value problem

$$\dot{x} = \begin{pmatrix} -1 & -1 & -2 \\ 1 & 1 & 1 \\ 2 & 1 & 3 \end{pmatrix} x + \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} e^t,$$

$$x(0) = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

OR

(b) (i)) Show that the given system of non linear differential equation is asymptotically stable at (0,0).

$$\begin{cases} \dfrac{dx}{dt} = -3x^3 - y \\ \dfrac{dy}{dt} = x^5 - 2y^3 \end{cases}$$

(ii)) Solve the initial value problem $\dot{x} = 3x - 2y, \dot{y} = 2y - x$ subject to initial condition $(x_0, y_0) = (3,4)$. Also draw the phase portrait of the above system.

M.A./M.Sc.-Math.-IVS-(CE-414)

2017

Time : **3** hours

Full Marks : **80**

The figures in the right hand margin indicate marks.
Answer from both the Sections as directed.

(ORDINARY DIFFERENTIAL EQUATIONS – II)

SECTION – A

1. *Answer any **four** of the following :* ***(4x4=16)***

(a) Plot the phase portrait and classify the fixed point of the linear system

$$\dot{x} = 3x - 4y, \dot{y} = x - y$$

(b) Prove that

$$\delta(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{ikx} dk$$

(c) Show that $P_n(x) = \frac{1}{n! 2^n}\frac{d^n}{dx^n}(x^2 - 1)^n$

(d) Show that $J_{1/2} = \sqrt{\frac{2}{\pi x}} \sin x$

(e) Let φ, ψ, χ be real valued continuous (or piecewise continuous functions on a real t interval $I: a \le t \le b$. Let $\chi(t) > 0$ on I, and suppose for $t \in I$ that

$$\varphi(t) \le \psi(t) + \int_a^t \chi(s)\varphi(s)ds$$

Prove that on I

$$\varphi(t) \le \psi(t) + \int_a^t \chi(s)\varphi(s) \exp\left(\int_s^t \chi(u)du\right) ds$$

(Turn over)

(f) Find e^{At} if $A = \begin{pmatrix} 1 & 1 & -1 \\ 2 & 3 & -4 \\ 4 & 1 & -4 \end{pmatrix}$

OR

2. *Answer all questions :* ***(2x8=16)***

(a) State strum's separation theorem.

(b) What is asymptotically stable?

(c) Define regular and singular point.

(d) Define Liapunov function.

(e) Show that a function of the form $ax^3 + bx^2y + cxy^2 + dy^3$ cannot be either positive definite or negative definite

(f) Write down the condition for which a spiral occurs in the system.

(g) Draw the phase portraits of the system of differential equation

$$\dot{x} = \begin{pmatrix} a & b \\ -b & a \end{pmatrix} x, \ a < 0, b < 0$$

(h) Show that the roots of the indicial of the differential equation

$$ty'' + y' - 4y = 0$$

are equal.

SECTION - B

Answer all questions ***(16x4=64)***

3. (a) Show that the equation

$$2(\sin t)y'' + (1 - t)y' - 2y = 0$$

has two solutions $y_1(t)$ and $y_2(t)$ of the form

$$y_1(t) = \sum_{n=0}^{\infty} a_n t^n, y_2(t) = t^{1/2} \sum_{n=0}^{\infty} b_n t^n$$

OR

(b) Find two linearly independent solutions of the Tchebycheff differential equation

$$(1 - t^2)y'' - 2ty' + \alpha^2 y = 0$$

where α is a constant.

4. (a) Find the Eigen values and corresponding Eigen functions of the Eigen value problem

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + \mu y = 0 \ (\mu > 0)$$

with the boundary condition $y'(1) = 0, y'(e^{2\pi}) = 0$.

OR

(b) Using Green's function method solve the boundary value problem

$$u'' + 3u' + 2 = \varphi$$
$$u(1) = 2u(0)$$
$$u'(1) = a$$

5. (a) State and prove the Strum's comparison theorem

OR

(b) State and prove Comparison theorem of Hille-Wintner