5. (a) Solve the problem by Kelley's cutting plane algorithm:

$$Mininmize\ 4x_1 + 5x_2$$

subject to

$$x_1^2 + 2x_1x_2 + 2x_2^2 \leq 4$$
$$-x_1^2 - x_2^2 + 4x_1 \geq 3$$

OR

(b) Solve the problem by Frank-Wolfe method

$$Mininmize\ (5x_1 + 2x_2)/(x_1 + 8x_2 + 1)$$

subject to

$$3x_1 + x_2 \geq 1$$
$$x_1 \geq 0, x_2 \geq 0$$

6. (a) A can is to be made in the form of right circular cylinder to contain at least V cubic inches of oil. What dimensions of the can will require the least amount of material?

OR

(b) A rectangular box, open at the top, is required to hold at least 256 cubic inches. Find the dimensions of the box for which the surface area is a minimum.

M.A./M.Sc.-Math.-IVS-(CE-404)

2017

Time : **3** hours

Full Marks : **80**

The figures in the right hand margin indicate marks.
Answer from both the Sections as directed.

(OPTIMIZATION TECHNIQUES – II)

SECTION – A

1. *Answer any **four** of the following :* *(4x4=16)*

(a) Prove that the quadratic program cannot have an unbounded solution when G is a positive matrix.

(b) Prove that the function $F(x) = -\sum_{i=1}^{s} x_i \ln(x_i/\sum_{i=1}^{s} x_i)$ is concave for $\boldsymbol{x} = (x_1, \ldots x_s)^T > \boldsymbol{0}$

(c) Prove that a point $x^0 \in \Omega_P$ is an optimal solution to Quadratic Program if and only if , for some $\lambda^0 \geq \boldsymbol{0}, (x^0, \lambda^0)$ is a saddle point of the Lagrangian $L(x, \lambda) = f(x) - \lambda^T(Ax - b)$, where $\lambda \geq 0$.

(d) Prove that a solution point x^* is K-T point of the problem

$$Minimize\ f(x)$$

subject to

$$\boldsymbol{Ax = b}$$
$$\boldsymbol{x \geq 0}$$

(e) Using Kelley's cutting plane method to solve the program:

$$Minimize\ z = -x_1 + x_2$$

subject to

$$x_1 + x_2 \leq 5$$
$$-x_1^2 - x_2^2 + 4x_1 - 3$$
$$x_1 \geq 0, x_2 \geq 0$$

(f) Solve the height of projectile problem.

(Turn over)

OR

2. *Answer **all** questions :* *(2x8=16)*

(a) Let $p_i > 0, q_i > 0,\ \sum_{i=1}^{r} p_i = \sum_{i=1}^{r} q_i$ then prove that $-\sum_{i=1}^{r} p_i \ln p_i \le -\sum_{i=1}^{r} q_i \ln q_i$

(b) State weak duality theorem.

(c) State strong duality theorem

(d) Let G be an $n \times n$ symmetric matrix, A an $m \times n$ matrix, and

$$M = \begin{pmatrix} G & A^T \\ -A & 0 \end{pmatrix}$$

Then prove that M is copositive if G is copositive

(e) Show how geometric programming can be used to test the consistency of the constraints

$$g_k(\boldsymbol{x}) \le 1,\ (k = 1, \dots, m)$$
$$\boldsymbol{x} > \boldsymbol{0}$$

(f) Define Kuhn-Tucker point.

(g) Let G be an $n \times n$ symmetric matrix. Further suppose the system

$$Mx + q \ge 0,\ \ x \ge 0$$

where $M = \begin{pmatrix} G & A^T \\ -A & 0 \end{pmatrix},\ q = \begin{pmatrix} c \\ b \end{pmatrix},\ x = \begin{pmatrix} z \\ \lambda \end{pmatrix},$ is inconsistent. Then prove that there exists a vector $\boldsymbol{d} \in \boldsymbol{R}^n, \boldsymbol{d} \ge \boldsymbol{0}$ such that $\boldsymbol{Ad} \le \boldsymbol{0}$.

(h) Define prototype primal geometric program.

(Turn over)

SECTION – B

*Answer **all** questions* *(16x4=64)*

3. (a) Apply Wolfe's method to the program:

Mininmize $f(x_1, x_2)$

$$= -10x_1 - 25x_2 + 10x_1^2 + x_2^2 + 4x_1x_2$$

subject to

$$x_1 + 2x_2 \le 10$$
$$x_1 + x_2 \le 9$$
$$x_1 \ge 0, x_2 \ge 0$$

OR

(b) Apply Fletcher's method to the program:

$$\textit{Mininmize } f(x_1, x_2) = \frac{1}{2}x_1^2 + 2x_2^2$$

sue bject to

$$3x_1 + 4x_2 \ge 13$$
$$x_1 \ge 0, x_2 \ge 0$$

4. (a) Use Lemke's complementary pivoting algorithm to solve the quadratic program:

$$\textit{Mininmize } 2x_1 + 4x_2 - x_1^2 - x_2^2 + 2x_1x_2$$

subject to

$$-x_1 + x_2 \le 1$$
$$x_1 - 2x_2 \le 4$$
$$x_1 \ge 0, x_2 \ge 0$$

OR

(b) Consider the linear complementarity problem of finding w and x satisfying

$$w - Mx = q,\ \ w^T x = 0,\ \ w \ge 0, x \ge 0$$

where

$$M = \begin{pmatrix} 1 & 0 & 2 \\ 3 & 2 & -1 \\ -2 & 1 & 0 \end{pmatrix},\ \ q = \begin{pmatrix} -1 \\ 2 \\ -3 \end{pmatrix}$$

(i) Is the matrix M copositive –plus?

(ii) Apply Lemke's algorithm to solve the problem.

(Turn over)