January, 2017

NUMERICAL ANALYSIS AND IT'S APPLICATION

Time : Three Hours] [*Maximum Marks* : 80

Answer from both the Sections as directed. The figures in the right-hand margin indicate marks.

SECTION-A

1. Answer any **four** of the following : 4×4

(*a*) Prove that $(1 + \Delta)(1 - \nabla) = 1$

(*b*) Find the cubic polynomial which takes the following values :

x :	0	1	2	3
$f(x)$:	1	2	1	10

(*c*) Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using Trapezoidal rule.

(*d*) Solve $y' = x+y$, $y(0) = 1$ by Taylor's Series Method. Hence find the values of y at $x = 0.1$.

(*e*) Using method of finite differences, find the sum of the following series:

$$1^2 + 2^2 + 3^2 + \ldots + n^2$$

(*f*) Derive the expression for second order derivative using central difference formulae.

OR

2. Answer all the questions from the following : 2×8

(*a*) Write down the formula for Hermite's interpolation.

(*b*) Evaluate $\Delta^2 (ab^x)$, interval of differencing is unity.

(*c*) What is maxima and minima of a Taulated function ?

(*d*) Prove that Euler's Method is the Runge-Kutta Method of the first order.

(*e*) Prove that $\Delta^n = \sigma^n E^{n/2}$

(*f*) Write Adams-Moulton corrector formula.

(*g*) Using Runge-Kutta Method of order 4, find y for $x = 0.1$, given that $\frac{dy}{dx} = xy + y^2$, $y(0) = 1$.

(*h*) Write the names of two self-starting methods to solve $y' = f(x, y)$, given $y(x_0) = y_0$.

SECTION-B

Answer **all** questions of the following : 16×4

3. (*a*) (*i*) Find the cubic polynomial which takes the following data :

x :	0	1	2	3
$f(x)$:	1	2	1	10

(*ii*) Apply Bessel's formula to obtain Y_{25}, given $Y_{20} = 2854$, $Y_{24} = 3162$

$$Y_{28} = 3544,\ Y_{32} = 3992$$

OR

(*b*) (*i*) Find the cubic splines for the following data :

x :	1	2	3
y :	–6	–1	16

(*ii*) The following table gives the values of x and y :

x :	1.2	2.1	2.8	4.1	4.9	6.2
y :	4.2	6.8	9.8	13.4	15.5	19.6

Find the value of x corresponding to $y = 12$, using Lagrange's technique.

4. (*a*) (*i*) Using Hermite's interpolation, find the value of $f(-0.5)$ from the following data:

x :	–1	0	1
$f(x)$:	1	1	3
$f'(x)$:	–5	1	7

(*ii*) Using inverse interpolation, find the real root of the equation $x^3 + x - 3 = 0$, which is close to 1.2.

OR

(*b*) (*i*) Evaluate $\int_0^2 \frac{x^2+2x+1}{1+(x+1)^4}\,dx$ by Gaussian 3-point formula.

(*ii*) Find the maximum and minimum value of y from the following table :

x :	–2	–1	0	1	2	3	4
y :	2	–0.25	0	–0.25	2	15.75	56

5. (*a*) (*i*) Given that :

x :	1.0	1.1	1.2	1.3	1.4	1.5	1.6
y :	7.989	8.403	8.781	9.129	9.451	9.750	10.031

Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 1.1$ and 1.6.

(*ii*) Evaluate $\int_0^1 \frac{dx}{1+x}$ taking 7 ordinates by applying Simpson's $\frac{3}{8}$th rule. Deduce the value of $\log_e 2$.

OR

(*b*) (*i*) Evaluate $\int_0^2 \frac{dx}{x^2+4}$ using Romberg's Method. Hence obtain the approximate value of Π.

(*ii*) Evaluate the double integral $\int_0^1 \int_0^1 \frac{dxdy}{1+x+y}$, by using the Simpson's rule with $h = k = 0.5$.

6. (*a*) (*i*) Given $\frac{dy}{dx} = x^2(1+y)$ and $y(1) = 1$, $y(1.1) = 1.233$, $y(1.2) = 1.548$, $y(1.3) = 1.979$. Evaluate $y(1.4)$ by Adams-Bashforth Method.

(*ii*) Using Milne's Method find $y(4.4)$ given $5xy' + y^2 - 2 = 0$ given $y(4) = 1$, $y(4.1) = 1.0049$, $y(4.2) = 1.0097$, $y(4.3) = 1.0143$; $y(4.4) = 1.0187$.

OR

(*b*) (*i*) Find an approximate series solution of the simultaneous equations $\frac{dx}{dt} = xy + 2t$, $\frac{dy}{dt} = 2ty + x$ subject to the initial conditions $x = 1$, $y = -1$, $t = 0$.

(*ii*) Apply Picard's Method to find the third approximations to the values of y and z, given that $\frac{dy}{dx} = z$, $\frac{dz}{dx} = x^3(y+z)$ given $y=1$, $z=\frac{1}{2}$ when $x=0$.