5. (*a*) Prove that a function f is of bounded variation on [a, b] if and only if f is difference of two monotone real-valued functions on $[a, b]$.

Or

(*b*) Prove that a function F is an indefinite integral if and only if it is absolutely continuous.

6. (*a*) State and prove Holder inequality.

Or

(*b*) State and prove Riesz-Fischer theorem.

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2017

Time : 3 hours

Full Marks : 80

The figures in the right-hand margin indicate marks.

Answer from both the Groups as directed.

(Abstract Measure)

GROUP—A

1. Answer any *four* of the following questions : 4×4

(*a*) Define outer measure. Show that m^* is translation invariant.

(*b*) Define the upper Riemann integral. Let f be bounded function defined on $[a, b]$. If f is Riemann integrable on $[a, b]$, then prove that it is measurable.

(*c*) Let f be a non-negative measurable function. Show that $\int f = 0 \Rightarrow f = 0$ a.e.

(Turn Over)

(d) If f be defined by

$$f(x)=\begin{cases}0, & x=0\\ x\sin\frac{1}{x}, & \text{if } x\neq 0\end{cases} \quad \text{then}$$

find $D^{-}f(0)$ and $D_{+}f(0)$.

(e) Define a norm in L^{∞}. Show that

$$\|f+g\|_{\infty}\leq\|f\|_{\infty}+\|g\|_{\infty}$$

Or

2. Answer *all* questions from the following : 2×8

(a) Define a measurable set.

(b) Give example of a continuous function g and a measurable function h such that $h \circ g$ is not measurable.

(c) Define convergence of a sequence $\langle f_n \rangle$ of measurable functions.

(d) Write the statement of Lebesgue convergence theorem.

(e) Show that $D^{+}[-f(x)]=-D_{+}f(x)$.

(f) What do you mean by Jensen inequality ?

(g) Define Banach space.

(h) What is Minkowski inequality ?

GROUP—B 16×4

3. (a) Let $\langle E_n \rangle$ be an infinite decreasing sequence of measurable sets. Let mE_1 is finite. Then prove that

$$m\left(\bigcap_{i=1}^{\infty}E_i\right)=\lim_{n\to\infty} mE_n$$

Or

(b) Prove that the collection of measurable sets is a σ-algebra.

4. (a) State and prove monotone convergence theorem.

Or

(b) If $\langle f_n \rangle$ is a sequence of nonnegative functions, then prove that

$$\int \underline{\lim}\, f_n \leq \underline{\lim} \int f_n$$