2016
(January)

Time : 3 hours

Full Marks : 80

The figures in the right-hand margin indicate marks.

Answer from both the Sections as per direction.

(MATHEMATICAL METHODS IN PHYSICS)

Section – A

1. Answer any **four** of the following : 4×4 = 16

   (a) Prove that $|z|^2$ is analytic.

   (b) Find the residue of the function $\frac{z^4}{z^2 + a^2}$ at $z = -ia$.

   (c) Prove that δ^i_j is a mixed tensor.

   (d) Explain the irreducible representation of SU(2) group.

   (e) Evaluate the Legendre polynomial $P_4(x)$.

(b) Prove $xJ'_n(x) = n J_n(x) - x J_{n+1}(x)$. Find the Fourier transform of :

$$Ft = \begin{cases} 1 - t^2 & |t| \le |\\ 0 & |t| > | \end{cases}$$

(f) Obtain the relation between Laplace and Fourier transform.

OR

2. Answer **all** questions from the following : 2×8 = 16

(a) Define Analytic function.

(b) What is residue ? Explain.

(c) What is Pseudo tensor ?

(d) What is a point group ?

(e) Define contravarient tensor.

(f) What is Fourier transform ?

(g) Define polynomial.

(h) Explain the importance of characters.

Section – B

Answer **all** questions : 16×4 = 64

3. (a) Derive an expression for the Laurent's expansion of a complex function. Evaluate $\int_0^1 \frac{x^2}{1+x^4}$ dx dy contour integration.

OR

(b) State and explain Residue Theorem. Find the residue of $f(z) = \frac{z^3}{(z-1)^4(z-2)(z-3)}$ at $z = 1$.

4. (a) Mention different types of tensors. If A^{ij} and B^k are tensors prove that Aji B^k; is also a tensor.

OR

(b) Explain the quotient law of tensor. Calculate the Christoffel symbols corresponding to the metric $ds^2 = (dx^1)^2 + G\,(dx^2)^2$, where G is a function of x^1, x^2.

5. (a) State and explain Cayley's theorem. Explain crystallographic point groups.

OR

(b) Obtain the group of symmetry operations of an equilateral triangle and derive the number of conjugate classes of the D_3 group.

6. (a) Obtain the orthogonality property of Bessel's Polynomial.

OR