2016

(December)

Time : **3** hours

Full Marks : **80**

The figures in the right hand margin indicate marks.
Answer from both the Sections as per direction.

(General Theory of Relativity)

Section –A

1. *Answer any **four** of the following :* *(4x4=16)*

   (a) A meson of mass M decays at rest into a μ-meson of mass m and a neutrino of zero mass. Find the expression for the kinetic energy of motion of the μ-meson (without the rest mass energy).

   (b) Obtain the expression for the covariant derivative of covariant vector V_μ.

   (c) Obtain the expression for gravitational red shift of light from an artificial satellite using time dilation in an arbitrary gravitational field.

   (d) Show that $R_{\lambda\mu\nu k} + R_{\lambda k\mu\nu} + R_{\lambda\nu k\mu} = 0$

   (e) Obtain a transformation equation for the affine connection.

   (f) Prove that the covariant derivative of metric tensor is zero.

OR

2. *Answer **all** the questions :* *(2x8=16)*

(a) Write down the covariant expressions for the Lagrangian and Hamiltonian of a relativistic free particle.

(b) Write down the equation of motion for a mass-less particle in an arbitrary gravitational field and an arbitrary co-ordinate system.

(c) Express the relation between g_{00} and the Newtonian potential.

(d) Prove that the Ricci tensor $R_{\mu k}$ is Symmetric.

(e) Prove that $g^{\lambda\mu} R_{\lambda\mu vk} = 0$

(f) Write the Einstein field equations in empty space.

(g) Show that $g^{\lambda\mu}\ g_{\mu v} = \delta^{\lambda}{}_{v}$

(h) Write the two important forms of Bianchi identity.

Section –B

*Answer **all** questions* *(16x4=64)*

3. (a) State the basic postulates of the special theory of relativity and derive the proper Lorentz transformations for (i) Space-time coordinates (ii) Momentum-energy of a particle.

OR

(b) Develop the relativistic kinematics for the elastic scattering of a particle when it collides with another particle at rest and obtain the expressions for (i) the angle of scattering (ii) angle of recoil (iii) energy transferred from the incident particle to the target particle.

4. (a) State and explain the principle of equivalence. Obtain the equation of motion for a particle in an arbitrary gravitational field and arbitrary coordinate system.

OR

(b) Establish the relation between affine connection and metric tensor. As a consequence of this relation show that the equation of the motion of a freely falling particle automatically maintains the form of proper time.

5. (a) Starting with the general equations of motion discuss how does one get contact with Newton's theory of gravitation.

OR

(b) Explain time dilation in an arbitrary gravitational field and discuss how the time dilation can be used to calculate the solar gravitational red shift.

6. (a) Derive Einstein's field equations in terms of energy-momentum tensor of matter and gravitational field.

OR

(b) Express the Riemann Christoffel Curvature tensor in terms of the affine connections. Prove that R^{λ} is the only the tensor that can be constructed from the metric tensor and its first and second derivatives.
