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Total Number of Pages: 02

**B.Tech**  
**BS1104**

**2<sup>nd</sup> Semester Back Examination 2015-16**

**MATHEMATICS-II**

**BRANCH: ALL**

**Time: 3 Hours**

**Max Marks: 70**

**Q.CODE: W276**

**Answer Question No.1 which is compulsory and any five from the rest.  
The figures in the right hand margin indicate marks.**

**Q1** Answer the following questions: **(2 x 10)**

- a) Find the Laplace transform of  $f(t) = 5^t$ .
- b) State convolution theorem.
- c) What is the period of the function  $f(x) = \cos\left(\frac{n \pi x}{L}\right)$ .
- d) Write the formula to find Fourier sine integral of any function.
- e) Prove that  $\beta(m, n) = \beta(n, m)$ , where  $\beta$  is a beta function.
- f) Find a vector which is perpendicular to the vectors  $-2\hat{i} + 5\hat{j} + 4\hat{k}$  and  $\hat{i} + 3\hat{j} + 2\hat{k}$ .
- g) The velocity vector of a fluid motion is given by  $\vec{v} = y\hat{i} - x\hat{j}$ . Check whether it is irrotational or incompressible.
- h) Write the formula to find the arc length of the curve  $C$ .
- i) Write the parametric representation of the sphere  $x^2 + y^2 + z^2 = 9$ .
- j) State Stokes's theorem.

**Q2** a) Solve the differential equations by using Laplace transform. **(5)**

$y'' + 4y = \sin 2t$ ,  $y(0) = 0$  and  $y'(0) = 1$ .

b) Find the inverse Laplace transform of the following: **(5)**

(i)  $\ln \frac{s^2 + 9}{s^2 + 1}$

(ii)  $\frac{s^3 - s^2 - s + 4}{s^4 - 5s^2 + 4}$ .

**P.T.O.**

**Q3 a)** Solve the integral equation  $y(t) = \sin 2t + \int_0^t \sin 2(t - \tau)y(\tau)d\tau$ . (5)

**b)** Find the Fourier series of the periodic function  $f(x) = 1 + x, 0 \leq x \leq 2\pi$ . (5)

**Q4 a)** Show that  $\int_0^\infty \frac{\cos x\omega}{1 + \omega^2}d\omega = \frac{\pi}{2}e^{-x}, x > 0$ . (5)

**b)** Find the Fourier Sine transformation of the following: (5)

$$f(x) = \begin{cases} \sin 2x, & 0 < x < \pi \\ 0, & x > \pi \end{cases}.$$

**Q5 a)** Prove that  $\text{div}(\vec{u} \times \vec{v}) = \vec{v} \cdot \text{curl} \vec{u} - \vec{u} \cdot \text{curl} \vec{v}$ . (5)

**b)** Find the directional derivative of  $f = 3xyz - xy^2$  at  $(1,2,2)$  in the direction of normal to the surface  $x^2 + y^2 - z^2 = 1$ , at  $(1,1,1)$ . (5)

**Q6 a)** Evaluate: (5)

$$\int_{(1,2,2)}^{(4,3,4)} yzdx + (xz - 2z)dy + (xy - 2y + 1)dz.$$

**b)** Find the moment of inertia of a lamina  $S : x^2 + y^2 = z^2, 0 \leq z \leq h$  of density 1 about the  $z$ -axis. (5)

**Q7 a)** Evaluate  $\int_C x^2 y dx - xy dy$ , where  $C$  is  $y = x$  and  $x + y = 2$ , from  $(0, 0)$  to  $(2,0)$ . (5)

**b)** Evaluate  $\int_0^{\pi/4} \int_x^{\pi/4} \frac{\sin y}{y} dy dx$ . (5)

**Q8 a)** Evaluate  $\iiint_S 5x dydz + 3y dx dz + 7z dx dy$  over an open box  $0 \leq x \leq 2,$  (5)

$0 \leq y \leq 2, 0 \leq z < 2$ .

**b)** Verify Green's theorem for  $\vec{F} = [5y, -xy]$  over a closed curve  $y = x$  and  $y = x^2$ . (5)

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