Registration No: | | | | | | | | | | |

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B.Tech.
15BS1104

2nd Semester Regular Examination 2015-16
MATHEMATICS - II
BRANCH: All
Time: 3 Hours
Max Marks: 100
Q.CODE: W275

Answer Part-A which is compulsory and any four from Part-B.
The figures in the right hand margin indicate marks.

Part – A (Answer all the questions)

Q1 Answer the following questions: ***multiple type or dash fill up type*** **(2 x 10)**

a) The inverse Laplace transform of $\frac{6s}{(s^2+9)^2}$ is __________.

b) The Laplace transform of $\sin t \times u(t-2\pi)$ is ________________.

c) The inverse Laplace transform of $\frac{3}{s^2+6s+18}$ is ____________.

d) The Fourier series of $\sin^2 x$ is ____________________________.

e) The smallest positive period of $\sin \pi x$ is _______.

f) If $\vec{F}(t) = t\,\hat{i} + \sin t\,\hat{j} + t^2\,\hat{k}$ then $\frac{d\vec{F}(t)}{dt} =$ ____________

g) If $f(x,y,z) = xyz$ then $\vec{\nabla} f =$ ____________________

h) For a scalar field f, Curl(gradf) = ____________

i) The value of β(2, 3) = _________________

j) The surface $\vec{r}(u,v) = [a\cos v,\ a\sin v,\ u]$ represents a ______________

Q2 Answer the following questions: ***Short answer type*** **(2 x 10)**

a) Represent the curve $x^2 + y^2 = 1,\ y = z$ parametrically.

b) Evaluate the divergence of $xyz(x\,\hat{i} + y\,\hat{j} + z\,\hat{k})$.

c) Evaluate the curl of $\sin y\,\hat{i} + \cos z\,\hat{j} + \hat{k}$

d) Find the surface normal to the surface $\vec{r}(u,v) = [u\cos v,\ u\sin v,\ cu]$

e) Find the direction of maximum increase of the function $\frac{z}{x^2+y^2}$ at (0, 1, 2).

f) Find the limits of the double integration $\iint f(x,y)\,dydx$ over the triangle with vertices (0 0), (1,0), and (1,1).

g) Find the Laplace transform of the function $f(t) = t^2,\ 0 < t < 1,\ 0$ otherwise.

h) Evaluate the convolution $e^t * e^{-t}$ by integration

i) Find the value to which the Fourier series of the function $f(x) = x,\ 0 < x < 2\pi$ will converge at $x = 2\pi$

j) Compute $\Gamma(-3.5)$

Part – B (Answer any four questions)

Q3 a) Use Laplace transform to solve $y'' + 6y' + 8y = e^{-3t} - e^{-5t},\ y(0)=0,\ y'(0)=0$. Provide the details of your work. **(10)**

b) Provide the inverse Laplace transform of $\ln\left(\dfrac{s+1}{s-1}\right)$ with details of all steps. **(5)**

Q4 a) Provide the details of the solution of the given equation. **(10)**
$y'' + 3y' + 2y = r(t),\ r(t) = 1\ if\ 0 < t < 1,\ 0\ if\ t > 1,\ y(0)=0,\ y'(0)=0$

b) Solve the integral equation $y(t) = 1 - \int_0^t (t-\tau)y(\tau)d\tau$. **(5)**

Q5 a) Using Green's theorem evaluate $\oint_C \vec{F}(r)\cdot d\vec{r}$ counter clockwise where C is the boundary of the triangle with vertices (0,0), (2,0) and (2,1). $\vec{F}(r) = [-e^{y},\ e^{x}]$. **(10)**

b) Show that the integral $\int_{(0,0,0)}^{(4,1,2)} (3y\,dx + 3x\,dy + 2z\,dz)$ is independent of path hence evaluate it. **(5)**

Q6 a) Using divergence theorem evaluate $\iint_S \vec{F}.\vec{n}dA$ where $\vec{F} = [\cos y, \sin x, \cos z]$, S is the surface of $x^2 + y^2 \le 4,\ |z| \le 2$. Provide the details of your work. **(10)**

b) Find the area enclosed by the cardioid r = a(1 – cosθ), 0≤ θ ≤ 2π. **(5)**

Q7 a) Work out in detail to find the Fourier series for the function $f(x) = 0,\ -2 < x < 0,\ f(x) = 2, 0 < x < 2,$ period = 4 **(10)**

b) Work out in detail to find the Fourier sine series for the function $f(x) = x,\ 0 < x < L.$ **(5)**

Q8 a) Work out in detail to find the Fourier integral of the function **(10)**
$f(x) = 0, x < 0,\ f(0) = \pi/2,\ \&\ f(x) = \pi e^{-x},\ x > 0$

b) Work out in detail to find the Fourier transform of the function $f(x)$ where $f(x) = 1,\ 0 < x < 1,\ \&\ f(x) = 0$ otherwise. **(5)**

Q9 a) Show that $\int_0^1 \left(\dfrac{x}{1-x^3}\right)^{1/2} dx = \dfrac{\sqrt{\pi}\,\Gamma\left(\frac{5}{6}\right)}{\Gamma\left(\frac{1}{3}\right)}$. Provide the details of your work. **(10)**

b) Find the scalar field f for which the vector field $grad\ f = \left[\dfrac{y}{z}, \dfrac{x}{z}, -\dfrac{xy}{z^2}\right]$. **(5)**