

January, 2017

**MATHEMATICAL METHODS IN  
PHYSICS**

*Time* : Three Hours]

[*Maximum Marks* : 80

Answer from both the Sections as directed. The figures in the right-hand margin indicate marks.

**SECTION-A**

1. Answer any four questions of the following : 4×4

(a) Show that the function  $\frac{1}{z^3 + 1}$  is analytic or not.

(b) Show that  $\int_0^{\infty} \frac{\sin x}{x} \cdot dx = \frac{\pi}{2}$  by contour integration.

(c) Show that the covariant derivative of  $\delta_j^i$  is zero.

(d) Explain the Dihedral Group.

(2)

(e) Show that

$$P_K^1(x) - 2x P_{K-1}^1(x) + P_{K-2}^1(x) = P_{K-1}(x).$$

(f) Find Laplace transform of  $\cosh^2 4t$ .

OR

2. Answer all questions from the following :

2×8

- Define Analytic Function.
- What is branch point and branch cut ?
- Define tensor.
- What is Covariant Tensor ?
- What is group and sub-group ?
- Define character.
- What is Laplace Transform ?
- Write one property of Fourier transform.

### SECTION-B

Answer all questions :

16×4

3. (a) Obtain the necessary and sufficient conditions of a function of a complex variable to be analytic. Prove that

$$I = \int_0^{2\pi} \frac{\cos 2\theta \, d\theta}{(5 + 4 \cos \theta)} = \frac{\pi}{6}.$$

OR

(3)

(b) State and prove Laurent's theorem. Explain the function  $f(z) = \frac{1}{z(z-1)}$  in terms of

Laurent's series.

4. (a) Give the transformation laws of tensor of 3rd order. Explain the quotient law of tensor.

OR

- (b) Explain fundamental tensors. Calculate the Christoffel symbols in cylindrical polar coordinates.
5. (a) Discuss the crystallographic point groups. Show that for a finite group  $G$ , every representation is equivalent to a unitary representation.

OR

- (b) Explain the character of representation. Show that the number of irreducible representations of an Abelian group equals to the number of group elements.
6. (a) Obtain the power series solution of Legendre's differential equation.

OR

- (b) Explain the Convolution theorem in Fourier transform. Evaluate Laplace transform of  $\sin^2(t)$ .