Registration No: | | | | | | | | | | |

Total Number of Pages: 2 **B.TECH**
PAM1A001

1^{st} Semester Regular Examination 2016-17
APPLIED MATHEMATICS- I
BRANCH(S): All
Time: 3 Hours
Max Marks: 100
Q.CODE: Y746

Answer Part-A which is compulsory and any four from Part-B.
The figures in the right hand margin indicate marks.

Part – A (Answer all the questions)

Q1 Answer the following questions: *fill in the blank* **(2 x 10)**

a) The curvature at any point on a circle with radius 10 is _______.

b) The asymptotes to the curve $xy^3 - x^2y + xy + 2x - y - 6 = 0$ which are parallel to the axis are ________.

c) The circular asymptote to the curve $r = \frac{2\theta}{\theta + 1}$ is _________

d) The number of arbitrary constants present in the solution of an ordinary differential equation depends on the _____ of the equation.

e) Integrating factor for the equation $xdy - ydx = 0$ is ______.

f) The maximum possible value of the rank of an 7×5 matrix is _______.

g) The dimension of the vector space generated by the vectors (2 0 1 0), (0 3 0 0) and (6 6 3 0) is ____.

h) A 2×2 matrix is both orthogonal and symmetric but it is not the identity matrix. The eigen values of this matrix are _______.

i) For the equation $y'' + ay' + by = e^x$ with roots of the auxiliary equation as 1 and 2 the particular solution will take the form _______.

j) The determinant of an orthogonal matrix is ______.

Q2 Answer the following questions: *Short answer type* **(2 x 10)**

a) Find the asymptotes to the curve $xy(x+y) = a(x^2 - a^2)$ which are parallel to the axis.

b) Find out the radius of curvature of the parabola $y = x^2$ at the origin.

c) Write down the sufficient (Lagrange's) conditions for a function of two variables to attain a maximum value.

d) Find the general solution of $y'' - 6y' + 9y = 0$

e) Find out the ordinary differential equation whose two independent solutions are x^2 and $x^2\ln x$.

f) Test whether the functions $x|x|$ and x^2 are linearly independent on the interval [–1, 1].

g) Find the Legendre polynomial $P_1(x)$ and $P_2(x)$ from the Rodrigues's formula.

h) Find the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{\sqrt{n^2+1}} x^n$

i) What are algebraic multiplicity and geometric multiplicity of an eigen value of a matrix?

j) What is a normal matrix? Show that a skew-symmetric matrix is a normal matrix.

Part – B (Answer any four questions)

Q3 a) Find all the asymptotes of the curve given by $x^3 - x^2y - xy^2 + y^3 + 2x^2 - 4y^2 + 2xy + x + y + 1 = 0$ **(10)**

b) Find the radius of curvature for the curve given parametrically by x = 3t and y = t^2 – 6 at t = 1. **(5)**

Q4 a) Find the extrema of the function $f(x, y) = x^3 + 3xy^2 - 3y^2 - 3x^2 + 4$ **(10)**

b) Due to global warming ice melts at a rate proportional to the amount present. If 2% of the original amount of ice melts in 100 years, how much will remain at the end of 1000 years? **(5)**

Q5 a) Solve $xy' + 2y = 3x^3 y^{\frac{4}{3}}$ **(10)**

b) Solve $ydx + (y^2 - x)dy = 0$ **(5)**

Q6 a) Solve $y'' - 2y' + (4\pi^2 + 1)y = 0,\ y(0) = -2,\ y'(0) = 6\pi - 2.$ **(10)**

b) Solve $y'' + y = \tan x.$ **(5)**

Q7 a) Solve $y'' + x^2 y = 0$ by series solution method **(10)**

b) Show that for the Legendre polynomial $P_n(x),\ P_n(-x) = (-1)^n P_n(x).$ **(5)**

Q8 a) Show that the system of linear equations 7x – 4y – 2z = – 6, 16x + 2y + z = 3 has solution and hence solve it using Gauss elimination method. **(10)**

b) Find the eigen values for the matrix $\begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$. **(5)**

Q9 a) Find out the type of conic section represented by the quadratic form $32x_1^2 - 60x_1x_2 + 7x_2^2 = -52$ by transforming it to principal axis. **(10)**

b) Show that the eigen values of a unitary matrix have absolute value 1. **(5)**