Registration No:

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B.TECH
15BS1101

1^{st} Semester Back Examination 2016-17
MATHEMATICS - I
BRANCH(S): ALL
Time: 3 Hours
Max Marks: 100
Q.CODE: Y747

Answer Part-A which is compulsory and any four from Part-B.
The figures in the right hand margin indicate marks.

Part – A (Answer all the questions)

Q1 Answer the following questions: **(2 x 10)**

a) A solution curve of the equation $xy' = 2y$, passing through (1,2), also passes through

I. (2,1) II) (4,24) III) (0,0) IV) (24,4)

b) The solution of the differential equation $y'' + y = 0$ satisfying the condition $y(0) = 1, y(\pi/2) = 2$,is ____________________.

c) The integrating factor of $\frac{dy}{dx} - y\tan x = x^2$ is ____________________.

d) The equation $M\,dx + N\,dy = 0$ is exact if _______________.

e) The value of the determinant $\begin{bmatrix} 1989 & 1990 & 1991 \\ 1992 & 1993 & 1994 \\ 1995 & 1996 & 1997 \end{bmatrix}$ is _______________.

f) If two eigen values of $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ are 3 and 15, then the third eigen value is ____________________.

g) The product of the eigen values of $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{bmatrix}$ is ________________.

h) The maximum value of the rank of a 4X5 matrix is _________.

i) The asymptote parallel to x –axis for the curve $f(x) = \frac{x-2}{x+1}$ is ___________.

j) $P_{2n+1}(0) =$ ____________, Where P_n is the Legendre polynomial.

Q2 Answer the following questions: **(2 x 10)**

a) Determine the order and degree of $(y'')^{3/2} + y' = x^2 y$.

b) Test the exactness of the differential equation
2sin2x sin h y dy = cos2x cosh 2y dx.

c) Formulate the differential equation for the R-C circuit with E = 100 volts, c = 0.25 farad, R = 200 ohms.

d) Define a linear differential equation of first order and give one example.

e) What do you mean by linearly independent vectors? Are the following vectors linearly independent? [2 -3], [3 6], [-1 4].

f) Explain the conditions for which a system of linear equations will posses more than one equation, unique solution & no solution?

g) Show that the determinant of a unitary matrix has absolute value 1.

h) Find the asymptote parallel to the co-ordinate axes of the curve
$xy^2 + x^2 y + 2xy - y + x + 2 = 0$.

i) What is the radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{(x-2)^n}{n!}$.

j) Find the value of $P_3(x)$ where P_3 is the Legendre polynomial.

Part – B (Answer any four questions)

Q3 a) Solve $(D^2 + 2D - 35)y = 12e^{5x} + 37\sin 5x$ by using method of undetermined coefficients. **(8)**

b) Find the general solution of the D.E $y'' - 2\sqrt{2}y' + 2y = 0$ **(7)**

Q4 a) Reduce to 1st order and solve the differential equation $xy'' + 2y' + xy = 0$ where $y = \frac{\sin x}{x}$ is a solution. **(8)**

b) Using method of variation of parameter, solve the following differential equation $\frac{d^2y}{dx^2} + 9y = \sec 3x$ **(7)**

Q5 a) Solve the differential equation $x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$ **(8)**

where y(0)=1.5 and $y'(0) = 1$.

b) Diagonalize the matrix $\begin{bmatrix} 3 & 5 & 3 \\ 0 & 4 & 6 \\ 0 & 0 & 1 \end{bmatrix}$. **(7)**

Q6 a) Solve the following linear system of equations by Gauss elimination method **(8)**

$x_1 - 2x_2 + 3x_3 = 0,\ -2x_1 + x_2 - 4x_3 = 3,\ 10x_2 + 5x_3 = 9,\ 6x_1 + 10x_2 = 0.$

b) Find a basis of eigen vectors of the matrix $\begin{bmatrix} 16 & 0 & 0 \\ 48 & -8 & 0 \\ 84 & -24 & 4 \end{bmatrix}$. **(7)**

Q7 a) Define the rank of the matrix and find the rank of the following matrix **(8)**

$$\begin{bmatrix} 2 & 0 & 1 & 3 \\ -2 & 4 & 6 & -3 \\ 1 & -4 & 1 & -5 \end{bmatrix}.$$

b) Find out what type of conic section is represented by the following quadratic form and transform it to principal axes $41x_1^2 - 24x_1x_2 + 34x_2^2 = 156$. **(7)**

Q8 a) Find the radius of curvature of the curve $x^{2/3} + y^{2/3} = a^{2/3}$ at the point $(0, a)$. **(8)**

b) Find the asymptotes of the following curve $(x-y)^2\ (x^2 + y^2) - 10(x-y)x^2 + 12y^2 + 2x + y = 0$. **(7)**

Q9 a) Prove that $(n+1)P_{n+1}(x) = (2n+1)xP_n(x) - nP_{n-1}(x)$. **(8)**

b) Show that $J_{-1/2}(x) = \sqrt{\dfrac{2}{\pi x}} \cos x$. **(7)**

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