Registration no: | | | | | | | | | | |

Total Number of Pages: 02 **B.Tech**

BS1101

1st Semester Back Examination 2016-17

MATHEMATICS - I

BRANCH(S): ALL

Time: 3 Hours

Max Marks: 70

Q.CODE: Y748

Answer Question No.1 which is compulsory and any five from the rest.

The figures in the right hand margin indicate marks.

Q1 **Answer the following questions:** **(2 x 10)**

a) Test the exactness of the differential equation 2sin2x sin h y dy = cos2x cosh 2y dx.

b) Define a linear differential equation. Give an example a linear differential equation of second order.

c) Determine the order and degree of $(y'')^{3/2} + y' = x^2 y$.

d) What do you mean by linearly independent vectors? Are the following vectors linearly independent? [2 3], [3 - 6], [-1 4].

e) Show that the main diagonal entries of a skew symmetric matrix are zero.

f) Find the asymptote parallel to the co-ordinate axes of the curve $xy^2 + x^2 y + 2xy - y + x + 2 = 0$.

g) Formulate the differential equation for the R-C circuit with E = 10 volts, c = 0.25 farad, R = 20 ohms.

h) Solve the differential equation $y' = y$ by power series method.

i) Find the value of $P_3(x)$ where P_3 is the Legendre polynomial.

j) What is the radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{(x-2)^n}{n!}$.

Q2 **a)** Reduce to 1st order and solve the differential equation $xy'' + 2y' + xy = 0$ where $y = \frac{\sin x}{x}$ is a solution. **(5)**

b) Solve : $y'' + 4y = 0, y(0) = 3, y(\pi/2) = -3$ **(5)**

Q3 **a)** Solve $(D^2 + 2D - 35)y = 12e^{5x} + 37\sin 5x$ by using method of undetermined coefficients. **(5)**

b) **(5)**

Solve the differential equation $x^2 \frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$

where y(0)=1.5 and $y'(0) = 1$.

Q4 **a)** Using method of variation of parameter, solve the following differential equation $\frac{d^2y}{dx^2} + 9y = \sec 3x$ **(5)**

b) Define the rank of the matrix and find the rank of the following matrix **(5)**

$$\begin{bmatrix} 2 & 0 & 1 & 3 \\ -2 & 4 & 6 & -3 \\ 1 & -4 & 1 & -5 \end{bmatrix}.$$

Q5 **a)** **(5)**

Find a basis of eigen vectors of the matrix $\begin{bmatrix} 16 & 0 & 0 \\ 48 & -8 & 0 \\ 84 & -24 & 4 \end{bmatrix}$.

b) Solve the following linear system of equations by Gauss elimination method **(5)**

$x_1 - 2x_2 + 3x_3 = 0, \ -2x_1 + x_2 - 4x_3 = 3, \ 10x_2 + 5x_3 = 9, \ 6x_1 + 10x_2 = 0.$

Q6 **a)** Find out what type of conic section is represented by the following quadratic form and transform it to principal axes **(5)**

$41x_1^2 - 24x_1x_2 + 34x_2^2 = 156$.

b) **(5)**

Diagonalize the matrix $\begin{bmatrix} 3 & 5 & 3 \\ 0 & 4 & 6 \\ 0 & 0 & 1 \end{bmatrix}$.

Q7 **a)** Find the radius of curvature of the curve $x^{2/3} + y^{2/3} = a^{2/3}$ at the point $(0, a)$. **(5)**

b) Find the asymptotes of the following curve **(5)**

$(x-y)^2 \ (x^2 + y^2) - 10(x-y)x^2 + 12y^2 + 2x + y = 0$.

Q8 **a)** **(5)**

Show that $\frac{d}{dx}(x^n J_n(x)) = x^n J_{n-1}(x)$.

b) **(5)**

Prove that $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n}\left[(x^2 - 1)^n\right]$.

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