**Registration no:** | | | | | | | | | | |

**Total Number of Pages: 03** **B.Tech**

**PEEI5402**

## 8th Semester Regular / Back Examination 2016-17
## OPTIMAL CONTROL
**BRANCH(S): EE, EEE**

**Time: 3 Hours**

**Max Marks: 70**

**Q.CODE: Z405**

**Answer Question No.1 which is compulsory and any five from the rest.**

**The figures in the right hand margin indicate marks.**

**Q1** **Answer the following questions:** **(2 x 10)**

**a)** Write down the expression for performance index for a finite time output regulator problem

**b)** What do you understand by the increment and the variation of a functional?

**c)** Find the variation of the functional

$$v(x_1, x_2) = \int_0^2 x_1 e^{-x_2} dt$$

Using the lemma $\delta v = \frac{d}{d\alpha} v(x + \alpha\, \delta x)\,|_{\alpha=0}$

**d)** What do you understand by transversality conditions?

**e)** What is 'Optimal Policy' in Dynamic Programming?

**f)** Write down the Algebraic Ricatti Equation and also the expression for the Optimal Control law.

**g)** Write down the three equations to be solved for solving the Optimal Control problem by Hamiltonian Method. What are the three equations called?

**h)** Differentiate between LQR and LQG controllers.

**i)** A dynamical system is modelled by the scalar differential equation

$$\dot{x} = ax + bu, \quad x(t_0) = x_0$$

with the associated cost index

$$J(t_0) = \frac{1}{2} f x^2(t_f) + \frac{1}{2}\int_{t_0}^{t_f} (qx^2(t) + ru^2(t))dt$$

Form the Hamiltonian function $H(t, x, u, \frac{\partial J^*}{\partial x})$ for the problem and determine $\partial H / \partial u$ .

**j)** What do you understand by the $H_\infty$norm of a transfer function?

**Q2** Find the points in the three-dimensional Euclidean space that extremise the function **(10)**

$$f(x_1, x_2, x_3) = x_1^2 + x_2^2 + x_3^2$$

and lie on the intersection of the surfaces

$$x_3 = x_1 x_2 + 5$$
$$x_1 + x_2 + x_3 = 1$$

**Q3 a)** Find the optimal control $u^*$ for the system **(7)**

$$\dot{X} = \begin{bmatrix} 0 & 1 \\ -10 & 0 \end{bmatrix} X + \begin{bmatrix} 0 \\ 10 \end{bmatrix} u$$

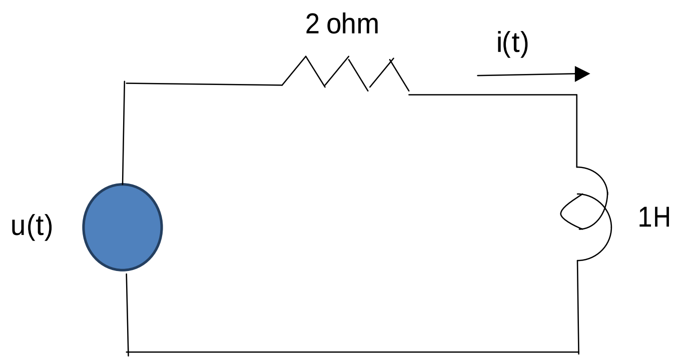
Which minimises the performance index

$J = \frac{1}{2}\int_0^2 u^2 dt$ Given $X(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $X(2) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

**b)** Write down the control equation, the state equation and the co-state equation for a system in terms of the Hamiltonian. **(3)**

**Q4 a)** For the circuit shown in figure, find the input voltage $u(t)$ that changes the current in the inductor from $i = 10A$ at $t = 0$ to $i = 0$ at $t = 1$ sec while minimizing the performance index **(6)**

$$J = \int_0^1 u^2(t)\, dt$$




**b)** The performance index to be minimised for a linear quadratic regulator is given by **(4)**

$$J = \frac{1}{2} X(t_f)^T S X(t_f) + \int_{t_0}^{t_f} [X^T(t) Q X(t) + u^T(t) R u(t)] dt$$

Explain the significance of each of the three terms and the nature and usefulness of the matrices in the above expression.

**Q5** Given, a second order plant **(10)**

$\dot{x}_1(t) = x_2(t),$ $x_1(0) = 2$

$\dot{x}_1(t) = -2x_1(t) + x_2(t) + u(t),$ $x_2(0) = -3$

And the performance index

$$J = \frac{1}{2}\int_0^\infty [2x_1^2(t) + 6x_1(t)x_2(t) + 5x_2^2(t) + u^2(t)] dt$$

Obtain the feedback control law using ARE.

**Q6 a)** Consider the following model of a dynamical system **(6)**

$$\dot{x} = 2x + u,$$

and the performance measure

$$J(x,u) = \int_0^\infty (x^2 + ru^2)\, dt$$

Find the value of r such that the optimal closed-loop system has pole at -3.

**b)** Discuss about Loop Transfer Recovery. **(5)**

**Q7** Find the closed loop optimal control for the first order system **(10)**

$$\dot{x}(t) = -2x(t) + u(t)$$

With the performance index

$$J = \int_0^\infty [x^2(t) + u^2(t)]dt$$

Assume that $J^* = fx^2(t)$.

**Q8** **Write short answer on any TWO:** **(5 x 2)**

**a)** Linear Fractional Transformation
**b)** Block diagram and Transfer function of LQG Controller
**c)** Discrete Ricatti Equation
**d)** Sub-Optimal Linear Regulator