Registration No: | | | | | | | | | | |

Total Number of Pages: 02 **M.TECH**

P2ECCC08

2nd Semester Regular Examination 2016-17

Statistical Signal Processing

BRANCH: COMMUNICATION ENGG, COMMUNICATION SYSTEMS, ELECTRO & COMM. ENGG, ELECTRO AND TELECOMMUNICATION ENGG, SIGNAL PROCESSING

Time: 3 Hours

Max Marks: 100

Q.CODE: Z849

Answer Question No.1 which is compulsory and any FOUR from the rest.
The figures in the right hand margin indicate marks.

Q1 Answer the following questions: ***Short answer type*** **(2 x 10)**

a) Evaluate the z-transform of a function $x(n)$ when it passes through a D-flip flop operating at 2 MHz clock frequency .

b) Evaluate and sketch the RoC of a function given as $\alpha^n u(n)$.

c) Which has better convergence: LMS or RLS algorithm? Justify.

d) Does the function $y(n) = kx(n) + C$, where k, C are constants and $x(n)$ is a fixed phase sinusoid, represent a random process? Justify.

e) The autocorrelation function of a function $x(n)$ is an ensemble average or a time average? Explain clearly.

f) What is stationary in a stationary random process? Explain.

g) How many poles and zeros characterize an AR process given as $AR(3)$?

h) What are the sizes of the autocorrelation function and the crosscorrelation function for an FIR Wiener filter that has five no.s of taps?.

i) Write down two salient features of a rectangular window.

j) Give the frequency response of a filter of length N having an impulse response given as $h(n) = \begin{cases} \frac{1}{N}\exp(jn\omega_i); & 0 \le n < N \\ 0 & \text{otherwise} \end{cases}$.

What is a reconstruction filter? Give its impulse response.

Q2 **a)** Find the autocorrelation sequence of a function whose PSD is given as $P_x(z) = \dfrac{-2z^2 + 5z - 2}{3z^2 + 10z + 3}$ **(10)**

b) Determine if the following two functions are valid autocorrelation sequences: (i) $r_x(k) = 3\delta(k) + 2\delta(k-1) + 2\delta(k+1)$ and (ii) $r_x(k) = \exp(jk\pi/4)$. **(10)**

Q3 **a)** How can you make the function given as $e^{-\frac{(x-a)^2}{2\sigma^2}}$ a probability distribution function? Compute its mean and the standard deviation **(10)**

b) Find and sketch the PSD for the following autocorrelation functions **(10)**

(i) $r_x(k)=\delta(k)+2(0.5)^{|k|}$ (ii) $r_x(k)=2\delta(k)+\cos\left(\frac{\pi k}{4}\right)$

Write comments on the results so obtained.

Q4 **a)** The input to an LTI system with impulse response $h(n)=\delta(n)+\frac{1}{2}\delta(n-1)+\frac{1}{4}\delta(n-2)$ is a zero mean WSS process with autocorrelation given as $r_x(k)=\left(\frac{1}{2}\right)^{|k|}$. Find out the variance and the autocorrelation of the output process. **(10)**

b) Unit variance white noise is applied to a filter having an impulse response given as $h(n)=\frac{2}{3}\left(-\frac{1}{3}\right)^n u(n)+\frac{1}{3}\left(-\frac{1}{3}\right)^{n-2} u(n-2)$.Find the PSD of the resulting output. Sketch the PSD. **(10)**

Show that a sufficient condition for a WSS process to be ergodic in the mean is that the autocovariance is absolutely summable, i.e.

$$\sum_{k=-\infty}^{\infty}|c_x(k)|<\infty$$

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Q5 **a)** Is the autocorrelation matrix of a random phase sinusoid positive definite? Examine first by finding the matrix. **(10)**

b) Establish the relationship between the power spectrum and the ensemble average of the Fourier magnitude of a given function $x(n)$. **(10)**

Q6 **a)** Design the optimum linear predictor for a real-valued AR(2) process defined as $x(n)$ = 0.9$x(n - 1)$ – 0.2$x(n - 2)$ + $w(n)$ where $w(n)$ is unit variance zero mean white noise. **(10)**

b) A noisy signal is expressed as $y(n)=x(n)+0.6x(n-1)+w(n)$ where $w(n)$ is a zero mean random white noise with unit variance. The signal $x(n)$ is a WSS AR(1) process with autocorrelation values given as $[4, 2, 1, 0.5]^T$. Design a non causal IIR Wiener filter that produces the MMSE of $x(n)$. . **(10)**

Q7 **a)** Evaluate the power spectrum of a random phase sinusoid embedded in a random white noise with unity variance. The phase is uniformly distributed in the interval of $[-\pi,\pi]$. Hence find out the mean value of the periodogram of this signal. **(10)**

b) Show that the LMS algorithm implements a low pass filter. Find out its convergence conditions. Hence, establish a relationship between the filter and the rate of convergence. **(10)**