Gandhi Institute of Engineering and Technology University, Odisha, Gunupur
(GIET UNIVERSITY)

M.Sc. (Fourth Semester – Regular/ Supplementary) Examinations, April – 2026

24MPCMA24001 –Functional Analysis

(Mathematics)

Time: 3 hrs Maximum: 60 Marks

Answer ALL questions

(The figures in the right hand margin indicate marks)

PART – A **(2 x 5 = 10 Marks)**

Q.1. Answer ***ALL*** questions

	Question	CO #	Blooms Level
a.	What do we mean by a convex set?	CO1	K1
b.	Define Normed Space with an example.	CO2	K1
c.	State Bounded Inverse theorem.	CO3	K3
d.	State the Polarization Identity	CO4	K3
e.	Define Unitary and Normal operator.	CO5	K3

PART – B **(10 x 5 = 50 Marks)**

Answer ***ALL*** the questions

	Question	Marks	CO #	Blooms Level
2. a.	Let X be a normed space, Y be a closed subspace of $X$and $Y \neq X$. Let r be a real number such that $0 < r < 1$. Then Prove that there exist some $x_r \in X$ such that $\|\|x_r\|\| = 1$and $r \leq dist(x_r, Y) \leq 1$.	5	CO1	K3
b.	Show that the norm$\|\| \ \ \|\|'$ is equivalent to the norm$\|\| \ \ \|\|$ if and only if there are $\alpha > 0, \beta > 0$, such that $\beta\|\|x\|\| \leq \|\|x\|\|' \leq \alpha\|\|x\|\|$, for all $x \in X$	5	CO1	K3
	(OR)			
c.	Let X be a normed space. Then show that the following conditions are equivalent. i. Every closed and bounded subset of X is compact. ii. The subset $x \in X: \|\|x\|\| \leq 1$ of X is compact. $iii.$ $X$is finite dimensional.	10	CO1	K3
3.a.	Let Xbe a normed space and Ybe a closed subspace of X. Then Prove that $X$is a Banach space if and only if $Y$and $X/Y$are Banach spaces with induced norm and quotient norm respectively.	10	CO2	K3
	(OR)			
b.	State and prove Closed Graph Theorem	10	CO2	K3
4.a.	State and prove Bessel's Inequality.	10	CO3	K3
	(OR)			
b.	Let H be a non-zero Hilbert space over K. Then Prove that the following conditions are equivalent (i) H has a countable orthonormal basis (ii) H is linearly isometric to K^n for some n. (iii) H is separable.	10	CO3	K3

		Marks	CO	K
5.a.	Let H be a Hilbert space and A$\in BL(H)$ Prove that Z(A)= R$(A^{*})^{\perp}$ and Z(A*)= R(A)$^{\perp}$ and A is injective if and only if R(A*) is dense in H, A* is injective if and only if R(A) is dense in H.	10	CO4	K3
	(OR)			
b.	Let H be a Hilbert space. and let A and B be being self-adjoint. then Prove that A+B is self-adjoint. Also Prove AB is self –adjoint if and only if A and B commute.	10	CO4	K3
6.a.	Let X be a Banach space and A ∈ BL(X) and $\|\|A^{p}\|\| < 1$ for some positive integer p, Then (I − A) is invertible and also $(I - A)^{-1} = \sum_{n=0}^{\infty} A_n$, Prove that $$\|\|(I - A)^{-1}\|\| \leq \frac{1 + \|\|A\|\| + \cdots + \|\|A^{p-1}\|\|}{1 - \|\|A^{p}\|\|}$$	10	CO5	K3
	(OR)			
b.	Let X be a reflexive normed space. Prove that (a) X is a Banach space and remains reflexive in any equivalent norm. (b) X' is reflexive. (c) Every closed subspace of X is reflexive.	10	CO6	K3

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