QP Code: R262G003 | Reg. No | | | | | | | | | | | AY 25

Gandhi Institute of Engineering and Technology University, Odisha, Gunupur
(GIET UNIVERSITY)

M.Sc. (Second Semester – Regular/Supplementary) Examinations, May – 2026

24MPCMA12001 – Abstract Algebra

(Mathematics)

Time: 3 hrs | Maximum: 60 Marks

Answer ALL questions

(The figures in the right hand margin indicate marks)

PART – A **(2 x 5 = 10 Marks)**

Q.1. Answer ***ALL*** questions		CO #	Blooms Level
a.	What do you mean by Quaternion group? Find inverse of each element of Quaternion group.	CO1	K2
b.	Differentiate between group homomorphism and isomorphism.	CO2	K1
c.	Define integral domain. Is $Z \times Q$ is an integral domain? Justify	CO3	K2
d.	What is minimal polynomial? Give an example.	CO4	K1
e.	What do you mean by Transcendental number and Algebraic number?	CO5	K1

PART – B **(10 x 5 = 50 Marks)**

Answer ***ALL*** the questions

			Marks	CO #	Blooms Level
2.	a.	If G is a subgroup, N a normal subgroup of G, then $\frac{G}{N}$ is also a group.	5	CO1	K1
	b.	Prove that if G is an abelian group, then for all $a, b \in G$ and all positive integers $n, (a.b)^n = a^n.b^n$.	5	CO1	k2
		(OR)			
	c.	Show that if every element of the group G is its own inverse, then G is abelian.	5	CO1	K2
	d.	State and prove Lagrange's theorem.	5	CO1	K1
3.	a.	The homomorphism ϕ of G into $\bar{G}$ with Kernel $K\phi$, is an isomorphism of G into $\bar{G}$ if and only if $K\phi = (e)$	10	CO2	K2
		(OR)			
	b.	Let ϕ be a homomorphism of G onto $\bar{G}$ with kernel K and let $\bar{N}$ be a normal subgroup of $\bar{G}$. $N = \{x \in G \mid \phi(x) \in \bar{N}\}$. Then $\frac{G}{N} \approx \frac{\bar{G}}{\bar{N}}$.	10	CO2	K2
4.	a.	If ϕ is a homomorphism of R into R', then a. $\phi(0) = 0$ b. $\phi(-a) = -\phi(a)$ for every $a \in R$.	5	CO3	K2
	b.	Prove that any field is an integral domain.	5	CO4	K2
		(OR)			
	c.	Let R be a commutative ring with unit element whose only ideals are (0) and R itself. Then R is a field.	5	CO3	K2
	d.	If R is a ring, then for all $a, b \in$ R show that (a) $(-1)a = -a$. (b) $a(-b) = (-a)b = -(ab)$.	5	CO4	K2
5.	a.	$a + ib$ is not a unit of $J[i]$ prove that $a^2 + b^2 > 1$.	5	CO4	K3
	b.	If p is a prime number of the form $4n + 1$, then we can solve the congruence	5	CO5	K3

$x^2 \equiv -1 \bmod p$.

(OR)

c.	Show that $[R: Q]$ is an infinite extension of Q, or $R = \infty$.	5	CO4	K3
d.	Minimal polynomial of any element is irreducible of F.	5	CO5	K3
6.a.	If $f(x), g(x)$ are two non zero elements of $F[x]$, then $\deg(f(x)g(x)) = \deg f(x) + \deg g(x)$	5	CO6	K2
b.	If $f(x)$ and $g(x)$ are primitive polynomials, then $f(x)g(x)$ is a primitive polynomial.	5	CO6	K2

(OR)

c.	Let $K\|F$ be any field extension. Then, $a \in K$ is algebraic over F iff $[F(a): F]$ is finite, i.e. $F(a)$ is a finite extension over F.	10	CO6	K3

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