

Gandhi Institute of Engineering and Technology University, Odisha, Gunupur
(GIET UNIVERSITY)



M.Sc. (Second Semester – Regular/Supplementary) Examinations, May – 2026
24MPCMA12003 – Partial Differential Equations
 (Mathematics)

Time: 3 hrs

Maximum: 60 Marks

Answer ALL questions
 (The figures in the right hand margin indicate marks)

PART – A**(2 x 5 = 10 Marks)**Q.1. Answer **ALL** questions

- Form a PDE by eliminating arbitrary constants from $z = ax + by + ab$
- Classify the PDE $u_{xx} - 2 \sin x u_{xy} - \cos^2 x u_{yy} - \cos x u_y = 0$
- Write various solutions of the diffusion equation $\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$.
- State the maximum-minimum principle related to diffusion equation
- Write the sufficient conditions for the existence of Laplace transform.

CO #	Blooms Level
CO1	K3
CO2	K2
CO3	K1
CO4	K1
CO5	K1

PART – B**(10 x 5 = 50 Marks)**Answer **ALL** the questions

- Form a PDE by eliminating the arbitrary function from $f(x + y + z, x^2 + y^2 + z^2) = 0$
 - Find the integral surface of the linear PDE $x(y^2 + z)p - y(x^2 + z)q = (x^2 - y^2)z$

Marks	CO #	Blooms Level
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4	CO1	K3
6	CO1	K1

(OR)

- Find the partial differential equation of the family of planes, the sum of whose x, y, z intercepts is equal to unity.
 - Find the complete integral of $x^2 p^2 + y^2 q^2 - 4 = 0$ using Charpit's method.
- Write different solutions of the Laplace's equation $u_{xx} + u_{yy} = 0$. Use it to solve the Dirichlet's problem for rectangle given by

$$\nabla^2 u = 0, 0 \leq x \leq a, 0 \leq y \leq b$$

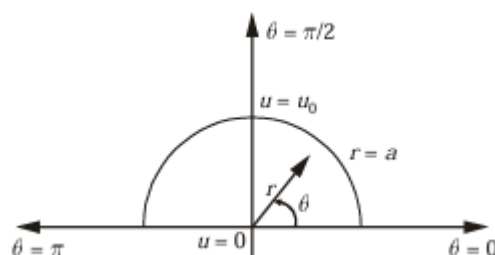
$$BCs': u(x, b) = u(a, y) = 0, u(0, y) = 0,$$

$$\text{and } u(x, 0) = f(x).$$

10	CO2	K2, K3
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(OR)

- Find the steady state temperature distribution in a semi-circular plate of radius a, insulated on both the faces with its curved boundary kept at a constant temperature U_0 and its bounding diameter kept at zero temperature as described as in the figure below



10	CO2	K1
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- 4.a Solve the one-dimensional diffusion equation in the region $0 \leq x \leq \pi, t \geq 0$
 . subject to the conditions
- (i) T remains finite as $t \rightarrow \infty$
 (ii) $T = 0$, if $x = 0$ and π for all t
 (iii) At $t = 0, T = \begin{cases} x, & 0 \leq x \leq \frac{\pi}{2} \\ \pi - x, & \frac{\pi}{2} \leq x \leq \pi \end{cases}$
- 10 CO3 K3
- (OR)
- b. The ends A and B of a rod, 10 cm in length, are kept at temperatures 0°C and 100°C until the steady state condition prevails. Suddenly the temperature at the end A is increased to 20°C , and the end B is decreased to 60°C . Find the temperature distribution in the rod at time t .
- 10 CO3 K3
- 5.a Find all the solutions of the one dimensional wave equation $u_{tt} - c^2 u_{xx} = 0$ using
 . variable separable method.
- 5 CO4 K2
- b. Obtain the solution of the wave equation $u_{tt} = c^2 u_{xx}$ under the following conditions
- (i) $u(0, t) = u(2, t) = 0$
 (ii) $u(x, 0) = \sin^3\left(\frac{\pi x}{2}\right)$
 (iii) $u_t(x, 0) = 0$
- 5 CO4 K1
- (OR)
- c. A uniform string of line density ρ is stretched to tension ρc^2 and executes a small transverse vibration in a plane through the undisturbed line of string. The ends $x = 0, L$ of the string are fixed. The string is at rest, with the point $x = b$ drawn aside through a small distance ε and released at time $t = 0$. Find an expression for the displacement $y(x, t)$.
- 10 CO4 K3
- 6.a Solve the following IBVP using the Laplace transform technique:
- PDE: $u_t = u_{xx}, \quad 0 < x < 1, t > 0$
 BCs': $u(0, t) = 1, u(1, t) = 1, t > 0$
 IC: $u(x, 0) = 1 + \sin \pi x, 0 < x < 1$.
- 10 CO5 K3
- (OR)
- b. Using Fourier transform method to solve the heat conduction equation given by
- $$k \frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, -\infty < x < \infty, t > 0$$
- 10 CO5 K3
- Subject to
- BCs': $u(x, t)$ and $u_x(x, t)$ both $\rightarrow 0$ as $|x| \rightarrow \infty$
 IC: $u(x, 0) = f(x), -\infty < x < \infty$

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