Gandhi Institute of Engineering and Technology University, Odisha, Gunupur
(GIET UNIVERSITY)

Ph.D. (First Semester) Examinations, December – 2025

23SPPEMT1012 - Graph Theory

(Mathematics)

Time: 3 hrs | Maximum: 70 Marks

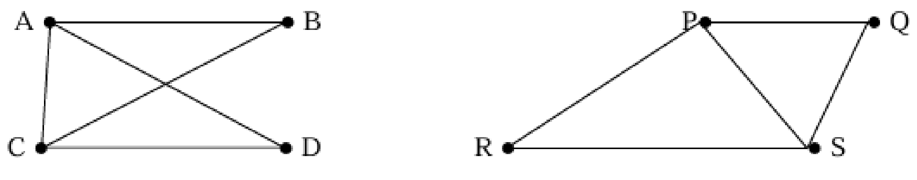
The figures in the right hand margin indicate marks.

Answer ANY FIVE Questions **(14 x 5 = 70 Marks)**

Marks

1.a. State and prove Hand shaking theorem. Also prove that the number of vertices with odd degree in a simple graph is even. 7

b. Verify whether the following two graphs are isomorphic or not. 7



G_1 G_2

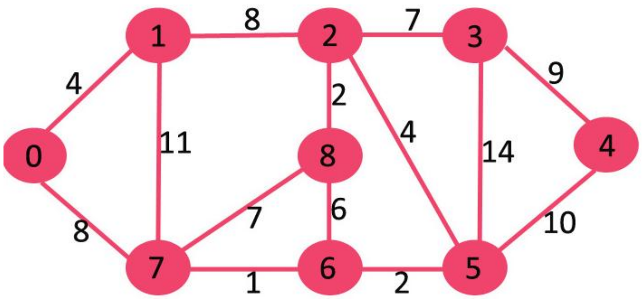
2.a. Define complete and complete bipartite graphs. Prove that the number of edges in a complete graph with 'n' vertices is $\frac{n(n-1)}{2}$ 7

b. For any graph G, prove that $\kappa(G) \leq \lambda(G) \leq \delta(G)$ 7

3.a. Prove the Euler formula for a graph to be Planar. Hence, prove that K_5 is non-planar. 8

b. Prove that every planar graph is 5-colorable. 6

4.a. Write the Prim's algorithm to find the minimal spanning tree. Use it to find the minimal spanning tree for the following graph 10



b. Prove that every tree is a bipartite graph. Also, state that which trees are complete bipartite graphs? 4

5.a. A graph is planar if and only if it has no subgraphs homeomorphic to K_5 or $K_{3,3}$,. 10

b. Prove that every tree has a center consisting of one point or two adjacent points. 4

6.a. Prove that the following statements are equivalent: 14

(i) G is a line graph

(ii) The lines of G can be partitioned into complete subgraphs in such a way that no point lies in more than two of the subgraphs.

(iii) G does not have $K_{1,3}$ as an induced subgraph, amd if two odd triangles have a common line then the subgraph induced by their points is K_4.

None of the nine graphs ia n induced subgraph of G.

7.a. Explain how a job sequencing problem can be solved using digraphs. 14

8.a. State and prove Heawood Map colouring theorem 8

b. For any graph G, prove that $\chi(G) \leq 1 + \delta(G)$ 6

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