Gandhi Institute of Engineering and Technology University, Odisha, Gunupur
(GIET UNIVERSITY)

M.Sc. (Third Semester – Regular) Examinations, December – 2025

24MPCMA23002 - TOPOLOGY

(Mathematics)

Time: 3 hrs Maximum: 60 Marks

Answer ALL questions
(The figures in the right-hand margin indicate marks)

PART – A **(2 x 5 = 10 Marks)**

Q.1. Answer ***ALL*** questions		CO #	Blooms Level
a.	Prove that closed intervals are closed sets.	CO1	K2
b.	Define base and sub base	CO2	K2
c.	Define connected and disconnected space.	CO3	K2
d.	Show that C = $\left\{\left(0,\frac{n}{n+1}\right) : n \in N\right\}$ is a cover of (0,1).	CO4	K2
e.	Show that every discrete space is a Hausdorff space.	CO5	K2

PART – B **(10 x 5 = 50 Marks)**

Answer ***ALL*** the questions		Marks	CO #	Blooms Level
2. a.	If T_1, T_2 are two topologies defined on the same set X, then $T_1 \cap T_2$ is also a topology on X, but $T_1 \cup T_2$ is not a topology on X	5	CO1	K2
b.	In any topological space, prove that I. $(A \cap B)^o = A^o \cap B^o$ II. ii. $(A \cup B)^o \supset A^o \cup B^o$	5	CO1	K3
	(OR)			
c.	In a topological space (X, T) if A, B ⊂ X, then prove that i. $b(A) = b(X - A) = \bar{A} - A^\circ = \bar{A} \cap \overline{(X - A)}$. ii. $\bar{A} = A^\circ \cup b(A)$. iii. $A^\circ = A - b(A)$ iv. $b(\bar{A}) \subset b(A)$ v. $b(A^\circ) \subset b(A)$	5	CO1	K2
d.	Let (X, T) be a topological space and $A \subset X$, A is closed iff $D(A) \subset A$	5	CO1	K3
3.a.	Show that A second countable space is always first countable space.	5	CO2	K2
b.	Show that A second countable space is always separable	5	CO2	K3
	(OR)			
c.	A second countable space is always first countable space.	5	CO2	K2
d.	Show that any open subspace of a separable space is separable.	5	CO2	K3
4.a.	Prove that the union of two non-empty separated subset of a topological space is disconnected.	5	CO3	K2
b.	If f and **g** are continuous real or complex functions defined on a topological	5	CO3	K3

	space **X,** then $f + g, \alpha f \text{ and } fg$ are also continuous. Furthermore, if $f \text{ and } g$ are real then $f \vee g \text{ and } f \vee g, \text{are continuous}$			
	(OR)			
c.	If A, B be separated sets of a topological space (X, T) then prove that • $A \cup B \text{ is closed } \Rightarrow A \text{ and } B \text{ are closed }.$ • $A \cup B \text{ is open } \Rightarrow A \text{ and } B \text{ are open}$	5	CO3	K2
d.	A map $f : X \longrightarrow y$ is continuous iff $f^{-1}(c)$ is closed in X for every closed set $C \subset Y$	5	CO3	K3
5.a.	A topological space is compact if every basic open cover has a finite sub cover.	5	CO4	K2
b.	Let (A, U) be a sub space of (X, T). Show that A is compact w.r.t to the topology U iff A is compact w.r.t the topology T on X	5	CO4	K3
	(OR)			
c.	A topological space (X, T) is compact iff each family of closed sets with finite intersection property has a non-empty inter section	10	CO4	K2
6.a.	A topological space (X, T) is a T_0-space iff for any distinct arbitrary points x, y of X, the closure of {x} and {y} are distinct	4	CO5	K2
b.	i. Show that every finite T_1-space is a discrete space. ii. Show that (X, T) is a T_4-space $\Rightarrow$ (X, T) is a T_2-space	6	CO5	K3
	(OR)			
c.	Show that the property of a space being a Hausdorff space is a hereditary property	5	CO5	K2
d.	A closed sub-space of a normal space is a normal space	5	CO5	K3

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