Reg. No | | | | | | | | | | | AY 24

Gandhi Institute of Engineering and Technology University, Odisha, Gunupur
(GIET UNIVERSITY)

M.Sc. (First Semester - Regular) Examinations, January – 2026

24MPCMA11001 – Linear Algebra
(Mathematics)

Time: 3 hrs | Maximum: 60 Marks

Answer ALL questions

(The figures in the right hand margin indicate marks)

PART – A **(2 x 5 = 10 Marks)**

Q.1. Answer ***ALL*** questions	CO #	Blooms Level
a. What is orthogonal decomposition?	CO4	K1
b. If the eigenvalues of a 3 by 3 matrix are 0,1,2 then find the trace and determinant of the matrix.	CO2	K2
c. Write down an example to show the rank and nullity of a matrix.	CO1	K2
d. What is triangle inequality? Proof it.	CO4	K2
e. Find the minimal polynomial of the matrix $A = \begin{bmatrix} 3 & -1 & 0 \\ 0 & 2 & 0 \\ 1 & -1 & 2 \end{bmatrix}$.	CO3	K2

PART – B **(10 x 5 = 50 Marks)**

Answer ***ALL*** the questions	Marks	CO #	Blooms Level
2. a. Let L be the set of all vectors of the form $(x, 2x, -3x, x)$ in V_4. Then L is a subspace of V_4.	5	CO1	K1
b. If V has a basis of n elements, then every other basis for V also has n elements.	5	CO1	K2
(OR)			
c. What do you mean by Algebraic multiplicity and Geometric multiplicity?	3	CO2	K1
d. Determine the algebraic and geometric multiplicity of $B = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$.	7	CO2	K2
3.a. Write the statement for the Caley-Hamilton Theorem and verified by it by example.	3	CO2	K1
b. Prove the Caley-Hamilton Theorem.	7	CO3	K1
(OR)			
c. Show that for any X, Hermitian form is real.	2	CO3	K1
d. Find the modal matrix and diagonalizes the matrix $C = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$.	8	CO3	K3
4.a. The set $\{v_1, v_2, v_3\}$ is L.D. iff v_1, v_2 and v_3 are coplanar.	5	CO1	K1
b. Determine the nature, index and signature of the quadratic form $x_1^2 + x_2^2 + x_3^2 + 2x_1x_2 - 6x_1x_3 - 8x_2x_3$.	5	CO3	K2
(OR)			
c. If f is finite dimensional and $v \neq 0 \in V$, then there is an element $f \in \hat{V}$ such that $f(v) \neq 0$.	10	CO4	K2

5.a.	If $\dim_F(V) = m$, then $\dim_F\big(\mathrm{Hom}(V, F)\big) = m$.	6	CO4	K2
b.	Let $B = \{(1,0,1), (1,2,1), (0,0,1)\}$ be a basis for $V = R^3$. Find the dual basis $\hat{V}$.	4	CO4	K3
	(OR)			
c.	Define norm linear space with example.	3	CO4	K3
d.	State and Prove Gram-Schmidt Orthogonalization Process.	7	CO4	K1
6.a.	What is orthogonalization? Give an example.	3	CO5	K1
b.	Write the derivation for Gram-Schmidt Orthogonalization Process. Show that (1/2,1/2,1/2,1/2), (1/2,1/2, -1/2, -1/2), (1/2, -1/2, -1/2,1/2), (-1/2,1/2, -1/2,1/2) is an Orthonormal bases of F^4.	7	CO5	K1
	(OR)			
c.	What do you mean by self-adjoint? Let $(V, <\,,>)$ be an inner product space and $T \in A(V)$ be self-adjoint. Then $T = 0$ iff $< Tx\,, x > = 0 \ \forall\, x \in V$.	10	CO5	K2

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