Reg. No | | | | | | | | | | | AY 25

Gandhi Institute of Engineering and Technology University, Odisha, Gunupur
(GIET UNIVERSITY)

M.Sc.(First Semester - Regular) Examinations, January – 2026

24MPCMA11002- Real Analysis

(Mathematics)

Time: 3 hrs   Maximum: 60 Marks

Answer ALL questions

(The figures in the right hand margin indicate marks)

PART – A   **(2 x 5 = 10 Marks)**

Q.1. Answer ***ALL*** questions	CO #	Blooms Level
a. Define the D'Alembert's Ratio Test.	CO1	K2
b. State Rolle's Theorem	CO2	K2
c. Define Riemann Integral	CO3	K2
d. State Young's theorem	CO4	K2
e. Define Taylor's theorem	CO5	K2

PART – B   **(10 x 5=50 Marks)**

Answer ***ALL*** the questions	Marks	CO #	Blooms Level
2. a. Let $X = x_n$ and $Y = y_n$ be a sequence of real numbers that converges to $x \ and \ y$, respectively, and let $c \in R$. Then the sequences $X + Y,\ X - Y,\ X * Y$, and cX converges to $x + y,\ x - y,\ xy \ and \ cx$ respectively	5	CO1	K3
b. Let $Y = (y_n)$ be a sequence defined inductively by $y_{n+1} = \frac{1}{4}(2y_n + 3)$ for $n \geq 1$ and $y_1 = 1$. Then prove tha $y = \frac{3}{2}$. .	5	CO1	K3
(OR)			
c. Let $e_n = \left(1 + \frac{1}{n}\right)^n$ for $n \in N$. Now show that the sequence $E = e_n$ is bounded and increasing; hence it is convergent.	5	CO1	K3
d. Test for convergence of the series $a_n = \frac{(2n)!}{4^n.(n!)^2}$	5	CO1	K3
3.a. State and prove Lagrange's Mean Value Theorem.	5	CO2	K3
b. State and prove Rolle's Theorem.	5	CO2	K3
(OR)			
c. Let I ⊆ R be an interval, let c∈I and let f: I→R and g: I→R be functions that are differentiable at c. Then a. If α∈R, then the function αf is differentiable at c and $(\alpha f)'(c) = \alpha f'(c)$ b. The function f+g is differentiable at c and $(f + g)'(c) = f'(c) + g'(c)$. c. The function fg is differentiable at c and $(fg)'(c) = f'(c), g(c) + f(c). g'(c)$. d. The function $\frac{f}{g}$ is differentiable at c, if g(c)≠0 and $\left(\frac{f}{g}\right)'(c) = \frac{f'(c),g(c) - f(c).g'(c)}{(g(c))^2}$.	10	CO2	K3

4.a.	Show that x^2 is integrable on any interval $[0, k]$ by using Riemann Integral	5	CO3	K3
b.	Show that $\int_1^2 (3x+1)\,dx = \frac{11}{2}$. By using Riemann Integral	5	CO3	K3
	(OR)			
c.	State and prove Darboux's Theorem.	5	CO3	K3
d.	Show that the function f defined by $f(x) = \begin{cases} 0, & \text{when x is rational} \\ 1, & \text{when x is irrational} \end{cases}$ is not integrable on any interval	5	CO3	K3
5.a.	Let (f_n) be a sequence of bounded functions on $A \subseteq R$. Then this sequence converges uniformly on *A* to a bounded function f if and only if for each $\epsilon \geq 0$ there is a number $H(\varepsilon)$ in N such that for all $m, n \geq H(\varepsilon)$ then $\parallel f_m - f_n \parallel_A \leq \varepsilon$	5	CO4	K3
b.	Let (f_n) be a sequence of functions in $R[a, b]$ and suppose that (f_n) converges uniformly on *[a, b] to f*. Then $f \in R[a, b]$ and $\int_a^b f = \lim_{n\to\infty} \int_a^b f_n$	5	CO4	K3
	(OR)			
c.	Let (f_n) be a sequence of bounded functions on $A \subseteq R$ converges uniformly *on A to f if and only if* $\parallel f_m - f \parallel_A \to 0$	5	CO4	K3
d.	Show that i. $\lim \left(\frac{x^2+nx}{n}\right) = x$ for $x \in R$ ii. Show that $\lim \frac{\sin(nx+n)}{\text{n}} = 0$ for $x \in R$	5	CO4	K3
6.a.	If *(a, b)* be a point of the domain of definition of a function *f* such that i. f_x is continuous at (a, b), ii. f_y is exist at (a, b), *then f is differentiable at (a, b).*	5	CO5	K3
b.	*Expand* $x^2y + 3y - 2$ *in powers of* $(x - 1)$ *and* $(y + 2)$. *Let us use Taylor's expansion with* $a = 1, b = -2$. *Then*	5	CO5	K3
	(OR)			
c.	*If* f_x *and* f_y *are both differentiable at a point (a, b) of the domain of definition of a function f, then* $f_{xy}(a, b) = f_{yx}(a, b)$	5	CO5	K3
d.	Prove that $f(x) = \begin{cases} \frac{x^3 - y^3}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$ is continuous at (0, 0).	5	CO5	K3

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