



**Gandhi Institute of Engineering and Technology University, Odisha, Gunupur  
(GIET UNIVERSITY)**

M.Sc. (Second Semester - Regular) Examinations, July - 2025

**24MPCMA12002 – Complex Analysis**

(Mathematics)

Time: 3 hrs

Maximum: 60 Marks

**(The figures in the right hand margin indicate marks)**

**PART – A**

**(2 x 5 = 10 Marks)**

Q.1. Answer **ALL** questions

- |                                                                 | CO # | Blooms Level |
|-----------------------------------------------------------------|------|--------------|
| a. What is the value of $\lim_{z \rightarrow 2i} (3x + iy^2)$ . | CO1  | K1           |
| b. Find the fixed points of $f(z) = z^2$ .                      | CO3  | K3           |
| c. Show that derivative of $(\bar{z})$ does not exist any where | CO2  | K2           |
| d. Evaluate $\int (\bar{z})^2 dz$ around the circle $ z =1$ .   | CO2  | K1           |
| e. Draw the figure of $ z  = a$ .                               | CO1  | K2           |

**PART – B**

**(10 x 5 = 50 Marks)**

Answer **ALL** the questions

- |                                                                                                                                                        | Marks | CO # | Blooms Level |
|--------------------------------------------------------------------------------------------------------------------------------------------------------|-------|------|--------------|
| 2. a. State and prove Cauchy-Reimann Equation.                                                                                                         | 5     | CO1  | K5           |
| b. Show that the function $f(z) = 4xy - x^3 + 3xy^2$ Harmonic and also find its Harmonic Conjugate                                                     | 5     | CO1  | K5           |
| (OR)                                                                                                                                                   |       |      |              |
| c. Prove that the function $ z^2 $ is continuous everywhere but nowhere differentiable except at origin                                                | 5     | CO1  | K2           |
| d. Find the bilinear transformation which maps the points $z_1=2, z_2=i, z_3=-2$ onto $w_1=1, w_2=i, w_3=-1$ respectively.                             | 5     | CO2  | K5           |
| 3.a. State and proof Liouville's theorem.                                                                                                              | 5     | CO3  | K6           |
| b. Using Cauchy integral formula evaluate $\int \frac{\cosh(\pi z)}{z(z-1)} dz$ over the circle $ z =2$ .                                              | 5     | CO2  | K5           |
| (OR)                                                                                                                                                   |       |      |              |
| c. Determine the number of zeros of the polynomial $P(z) = z^{10} - 6z^7 + 3z^3 + 1$ in $ z  < 1$ .                                                    | 5     | CO2  | K1           |
| d. State and proof Cauchy integral theorem.                                                                                                            | 5     | CO2  | K2           |
| 4.a. Find the radii of convergence (i) $\sum \frac{n!}{n^n} z^n$ (ii) $\sum \frac{z^n}{n^2}$ .                                                         | 5     | CO5  | K3           |
| b. Prove that $\int_{-\infty}^{\infty} \frac{x^4}{x^6-1} dx = \frac{\pi\sqrt{3}}{6}$ .                                                                 | 5     | CO4  | K2           |
| (OR)                                                                                                                                                   |       |      |              |
| c. If $z, w$ be two complex numbers such that $z\bar{w} \neq 1$ and $ z <1,  w <1$ . Prove that $\left  \frac{z-w}{1-\bar{z}w} \right  < 1$ .          | 5     | CO3  | K3           |
| d. Determine the region of the $w$ - plane into which each of following is mapped by the transformation $w=iz+1$ , (i) $x>0; 0<y<2$ of the $z$ -plane. | 5     | CO2  | K2           |
| 5.a. Expand $f(z) = \frac{1}{z}$ in a Taylor's series about $z = 1$ .                                                                                  | 4     | CO6  | K3           |

- b. Evaluate  $\int_0^{2\pi} \frac{\cos 2\theta}{5+4\cos\theta} d\theta$ . 6 CO6 K3
- (OR)
- c. If  $f(z) = \frac{z+4}{(z+3)(z-1)^2}$ , find Laurent's series expand in , 5 CO6 K3  
a)  $0 < |z-1| < 4$  b)  $|z-1| > 4$
- d. State and prove Maximum Modules theorem. 5 CO4 K2
- 6.a. Prove that  $\int_0^\infty \frac{\cos mx}{a^2+x^2} dx = \frac{\pi}{2a} e^{-ma}$ ,  $m \geq 0$ . 5 CO4 K3
- b. Find the poles of  $f(z) = \frac{z^2+4}{z^3+2z^2+2z}$  and determine the residue at pole 5 CO5 K3
- (OR)
- c. Find the singularities and write which type of singularity of given function 5 CO6 K3  
 $f(z) = \frac{z-\sin z}{z^3}$ .
- d. State and prove Cauchy's Residue theorem. 5 C04 K2

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