QP Code: R252G027 | Reg. No | | | | | | | | | | | AY 24

Gandhi Institute of Engineering and Technology University, Odisha, Gunupur
(GIET UNIVERSITY)

M.Sc. (Second Semester - Regular) Examinations, July – 2025

24MPCMA12001 – Abstract Algebra

(Mathematics)

Time: 3 hrs — Maximum: 60 Marks

Answer ALL questions

(The figures in the right hand margin indicate marks)

PART – A **(2 x 5 = 10 Marks)**

Q.1. Answer ***ALL*** questions		CO #	Blooms Level
a.	What is Commutative group. Provide an example of Commutative group.	CO1	K1
b.	Differentiate between group homomorphism and isomorphism.	CO2	K1
c.	Let G be the group of integers under addition. A mapping $\phi: G \to G$ is defined by $\phi(x) = cx$ for all $x \in G, c \in Z$. Then show that ϕ is a homomorphism.	CO3	K2
d.	Define Eisenstein's Criterion and give an example.	CO4	K2
e.	What do you mean by extension field? Justify your answer with an example.	CO5	K2

PART – B **(10 x 5 = 50 Marks)**

<u>Answer ***ALL*** the questions</u>		Marks	CO #	Blooms Level
2. a.	State and Prove Lagrange's Theorem.	7	CO1	K2
b.	Is the set of integers is a cyclic group w.r.t addition? If yes then find its generator.	3	CO1	K2
	(OR)			
c.	N is a normal subgroup of G if and only if $gNg^{-1} = N$ for every $g \in G$.	7	CO1	K2
d.	Define Euler's ϕ function. Find $\phi(10)$.	3	CO1	K2
3.a.	Show that if every element of the group G is its own inverse, then G is abelian.	6	CO2	K2
b.	If every $x \in R$ satisfies $x^2 = x$. Prove that R must be commutative.	4	CO1	K2
	(OR)			
c.	Let ϕ be a homomorphism of G onto $\bar{G}$ with kernel K and let $\bar{N}$ be a normal subgroup of $\bar{G}$. $N = \{x \in G \mid \phi(x) \in \bar{N}\}$. Then $\frac{G}{N} \approx \frac{\bar{G}}{\bar{N}}$.	10	CO2	K2
4.a.	Show that the commutative Ring D is an integral domain iff for $a, b, c \in D$ with $a \neq 0$, the relation $ab = ac$ implies that $b = c$.	5	CO3	K2
b.	If R is a ring, then for all $a, b \in R$ show that, $(-1)a = -a$.	5	CO3	K2
	(OR)			
c.	If U is an ideal of a ring R, then $\frac{R}{U}$ is a ring and is a homomorphic image of R.	10	CO3	K2
5.a.	Show that the commutative Ring D is an integral domain iff for $a, b, c \in D$ with $a \neq 0$, the relation $ab = ac$ implies that $b = c$.	5	CO4	K3
b.	If R is a ring, then for all $a, b \in R$ show that, $(-1)a = -a$.	5	CO3	K1
	(OR)			
c.	Show that $F[x]$ is an Euclidean ring.	6	CO4	K2
d.	The homomorphism ϕ of R into R' is an isomorphism iff $I(\phi) = (0)$.	4	CO3	K3

6.a.	Minimal polynomial of any element is irreducible of F.	5	CO5	K1
b.	If $P(x) \in F[x]$ be a minimal polynomial of α and $f(x) \in F[x]$ be any other polynomial such that $f(\alpha) = 0$, then $P(x) \mid f(x)$.	5	CO5	K3
	(OR)			
c.	Minimal polynomial of an element is unique.	5	CO5	K1
d.	Every finite extension is an algebraic extension.	5	CO5	K3

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