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GANDHI INSTITUTE OF ENGINEERING AND TECHNOLOGY UNIVERSITY, ODISHA, GUNUPUR

(GIET UNIVERSITY)

Ph.D. (First Semester-Winter) Examinations, June – 2025

23WPPEMT1011 – Advanced Mathematics (Mathematics)

Time: 3 hrs Maximum: 70 Marks

The figures in the right hand margin indicate marks.

Answer *ANY FIVE* Questions. (14 x 5 = 70 Marks) **Marks**

1. Prove that the set of linear differential operators with constant coefficients forms a non-commutative ring under composition. Provide examples to illustrate non-commutativity. 14

2. a Find the eigenvalues and corresponding eigen functions of the periodic Strum-Liouville problem $y'' + \lambda y = 0, \quad y(0) = 0\,, \quad y'(L) = 0$ 7

b. Find the Fourier series of f(x) = x, $-\pi < x < \pi$. 7

3. a. Find the Fourier cosine integral and Fourier sine integral of $f(x) = \begin{cases} \sin x\,, \text{if } 0 \le x \le \pi \\ 0, \text{ if } x > \pi \end{cases}$ 7

b. Solve the linear difference equation by Using the Z-transform 7

$y_{n+2} - 3y_{n+1} + 2y_n = 0, \quad y(0) = 1, \quad y(1) = 2.$

4.a Apply Runge- Kutta method (Fourth order) to find the approximate value of $y(0.2)$ given that $\frac{dy}{dx} = y - x^2 + 1\,, \quad y(0) = 0.5, at\ h = 0.2$ 7

b. Apply Taylor's series method to find the approximate value of $y(0.1)$ given that 7

$\frac{dy}{dx} = 3x + y^2\,, \quad y(0) = 1$

5. Solve the PDE by applying numerically method $u_t = 2u_{xx,}$ 14

BCs: $u(0,t) = 10,\ u(6,t) = 18,\ u(x,0) = \frac{x^2}{2}$

6. Solve the ODE $y'' + 4y' + 4y = sint,\ y(0) = 0,\ y'(0) = 1$ using Laplace transforms 14

7. Solve the wave equation $u_{tt} = C^2 u_{xx,}$ with initial and boundary conditions using Laplace transforms. 14

8. Use the Fourier transform to solve the initial value problem for the heat equation 14

$u_t = \alpha^2 u_{xx}\,, \qquad -\infty < x < \infty$

---End of Paper---