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GANDHI INSTITUTE OF ENGINEERING AND TECHNOLOGY UNIVERSITY, ODISHA, GUNUPUR

(GIET UNIVERSITY)

Ph.D. (First Semester-Winter) Examinations, June – 2025

23WPPEMT1014 – Advanced Complex Analysis

(Mathematics)

Time: 3 hrs Maximum: 70 Marks

The figures in the right hand margin indicate marks.

	Answer *ANY FIVE* Questions. (14 x 5 = 70 Marks)	**Marks**
1.a.	State and prove Schwarz's Lemma. Also, derive a bound for a holomorphic function mapping the unit disk into itself.	7
b.	Explain Picard's Theorem for essential singularities. Give an example of a function that illustrates this behavior.	7
2.	Define analytic continuation. Prove that if two analytic functions agree on a set having a limit point inside a domain, then they agree everywhere on that domain.	7
b.	Explain the concept of analytic continuation via reflection across a real-analytic boundary. Apply it to a function defined on a half-disk.	7
3 a.	State and explain Weierstrass Factorization Theorem. Use it to construct an entire function with prescribed zeros..	7
b.	Define infinite product of analytic functions. Prove that the product $\prod(1 - z^n)$ converges uniformly on compact subsets of the unit disk.	7
4.a.	Prove that the Gamma function satisfies the identity $\Gamma(z+1) = z\Gamma(z)$. Derive the reflection formula for $\Gamma(z)$.	7
b.	State Jensen's formula. Use it to relate the growth of a function to the distribution of its zeros in the disk.	7
5.a.	State and prove the Open Mapping Theorem. Explain its significance in complex function theory.	7
b.	What is a normal family? State Montel's theorem and explain its role in the proof of the Riemann Mapping Theorem.	7
6.	Explain the concept of genus of an entire function. Give an example and determine the order and genus of the exponential function.	14
7.a.	State and discuss the Bloch–Landau Theorem. How does it relate to the size of images under holomorphic functions?	7
b.	State and prove the Poisson integral formula. Apply it to solve the Dirichlet problem for the unit disk.	7
8 a.	Prove that a uniform limit of analytic functions is analytic. Apply this to justify the convergence of an infinite product of analytic functions.	7
b.	Prove Hurwitz's Theorem. Use it to show that the limit of a sequence of univalent functions is either univalent or constant under uniform convergence.	7

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