

**ANDHI INSTITUTE OF ENGINEERING AND TECHNOLOGY UNIVERSITY, ODISHA, GUNUPUR
(GIET UNIVERSITY)**

Ph. D. (Second Semester – Regular) Examinations, May – 2025

**23SPPEMT2011 – Advanced Graph Theory
(Mathematics)**



Time: 3 hrs

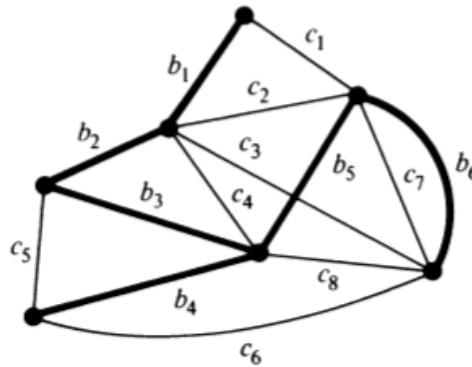
Maximum: 70 Marks

The figures in the right-hand margin indicate marks.

Answer ANY FIVE Questions

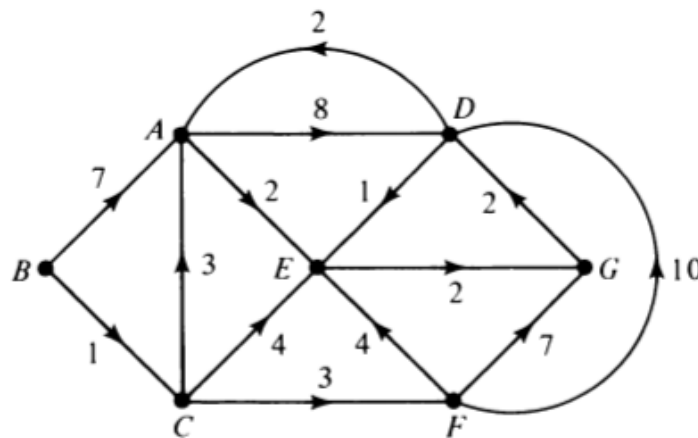
(14 x 5 = 70)

- 1.a. Prove that a connected graph with n vertices is a tree if and only if it has $n - 1$ edges. 6
- b. Find two spanning trees for the following graph



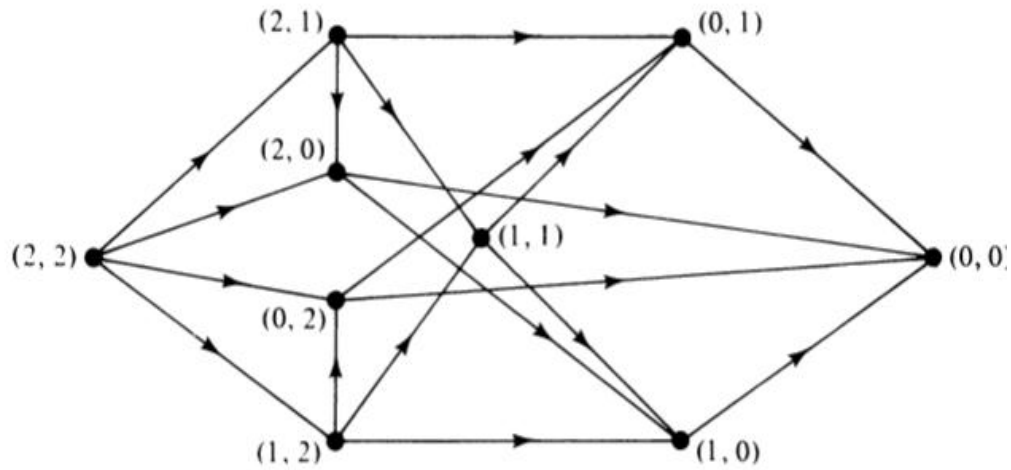
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- 2.a. Prove that the ring sum of two circuits in a graph G has either a circuit or an edge disjoint union of circuits. 7
- b. Prove that the vector space of a graph, the circuit subspace and the cut-subspace are orthogonal to each other. 7
- 3.a. Prove that subspaces W_T and W_S are orthogonal complements if and only if $\dim(W_T \cap W_S) = 0$ 7
- b. Find the number of simple graphs with $n(= 3,4,5)$ vertices and r edges (for all possible values of r) using Polya's counting theorem. 7
- 4.a. Explain Dijkstra's algorithm and employ it to find the shortest distance from vertex B to vertex G in the following digraph:

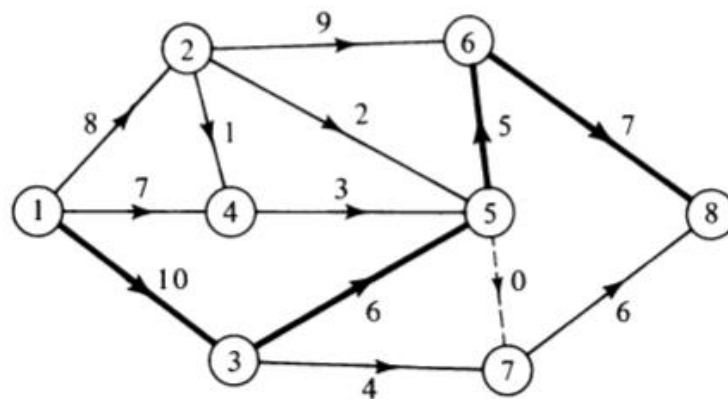


14

- 4.a. Describe an algorithm to find all directed circuits of a digraph with suitable example. 7
- b. Explain how a graph is presented and stored in computers. 7
- 5.a. Prove that every acyclic digraph has unique kernel. 7
- b. Explain uses of graphs in Coding theory. 7
- 6.a. Find an acyclic digraph in the following graph : 14

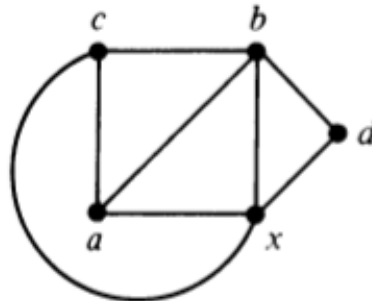


7. Describe an algorithm to find critical path and hence project duration of an activity network and employ the same to find CPM in the following graph



14

- 8.a. Describe DFS algorithm and illustrate how it is applied to the following graph



7

- b. Prove that there are n^{n-2} labeled trees with n vertices ($n \geq 2$)

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