Reg. No | | | | | | | | | | |   AY - 23

**GANDHI INSTITUTE OF ENGINEERING AND TECHNOLOGY UNIVERSITY, ODISHA, GUNUPUR**

**(GIET UNIVERSITY)**

M. Sc. (Fourth Semester) Regular Examinations, April 2025

# 22MTPC403 - Functional Analysis

(Mathematics )

Time: 3 hrs  Maximum: 70 Marks

**(The figures in the right hand margin indicate marks.)**

**PART – A** **(2 x 10 = 20 Marks)**

Q.1. Answer ***ALL*** questions

| | Question | CO # | Blooms Level |
|---|---|---|---|
| a. | What do we mean by a convex set? | CO1 | K1 |
| b. | What is the condition for a mapping $F: X \to Y$ to be a Homeomorphism? | CO1 | K2 |
| c. | Define Normed Space with an example. | CO2 | K2 |
| d. | Define the Banach space. | CO2 | K3 |
| e. | Define the Interior point and the limit point of a space E. | CO3 | K3 |
| f. | State the Bounded Inverse theorem. | CO3 | K3 |
| g. | State the Polarization Identity | CO4 | K3 |
| h. | Define Schwarz Inequality | CO4 | K3 |
| i. | State the Riesz representation theorem | CO5 | K3 |
| j. | Define Unitary and Normal operator | CO5 | K3 |

**PART – B** **(10 x 5 = 50 Marks)**

Answer ***ANY FIVE*** questions

| | Question | Marks | CO # | Blooms Level |
|---|---|---|---|---|
| 2. a. | Show that the norm$\|\| \ \|\|'$ is equivalent to the norm$\|\| \ \|\|$ if and only if there are $\alpha > 0, \beta > 0$, such that $\beta\|\|x\|\| \leq \|\|x\|\|' \leq \alpha\|\|x\|\|$, for all $x \in X$ | 5 | CO1 | K2 |
| b. | Let $X$ be a normed space, $Y$ be a closed subspace of $X$and $Y \neq X$. Let $r$ be a real number such that $0 < r < 1$. Then there exist some $x_r \in X$ such that $\|\|x_r\|\| = 1$and $r \leq dist(x_r, Y) \leq 1$. | 5 | CO1 | K2 |
| 3. | Let $X$ be a normed space. Then show that the following conditions are equivalent.<br>i. Every closed and bounded subset of $X$ is compact.<br>ii. The subset $x \in X: \|\|x\|\| \leq 1$ of $X$ is compact.<br>$iii.$ $X$is finite dimensional. | 10 | CO2 | K2 |
| 4. a. | Let $X$ and $Y$ be normed spaces. Then show that, if $X$ is finite dimensional, then every linear map from $X$ to $Y$ is continuous. Conversely, if $X$ is finite dimensional and $Y \neq \{0\}$, then there is a discontinuous linear map from $X$ to $Y$. | 5 | CO2 | K2 |
| b. | State and Prove Hahn-Banach Extension theorem. | 5 | CO2 | K2 |
| 5.a. | Let $X$ and $Y$ be a normed space and $F: X \to Y$ be a linear map such that the range $R(F)$ of $F$ is finte dimensional. Then prove that $F$ is continuous if and only if the zero space $Z(F)$ of $F$ is closed in $X$ | 5 | CO3 | K2 |
| b. | Prove that, a Normed space $X$ is a Banach space if and only if every absolutely summable series of elements in $X$ is summablein $X$. | 5 | CO3 | K2 |

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|---|---|---|---|---|
| 6. | Let H be a nonzero Hilbert space over K. Then the following conditions are equivalent<br>(i) H has a countable orthonormal basis<br>(ii) H is linearly isometric to $K^n$ for some n, .<br>(iii) H is separable. | 10 | CO4 | K3 |
| 7.a. | State and Prove Riesz-Fischer theorem. | 5 | CO4 | K3 |
| b. | State and Prove Gram- Schmidt orthonormalization. | 5 | CO4 | K3 |
| 8. a. | Let H be a Hilbert space. consider A, B$\in BL(H) and\ \ k \in k. Then\ Prove\ that$<br>(a) (A+B) *=A*+B* (b), (KA*) =$\bar{k}$A*, (c) (AB)*=B*A*, (d) (A*) *=A. | 5 | CO5 | K3 |
| b. | Let H be a Hilbert space and A and B be normal. Prove that If A commutes with B* B commutes with A* then A+B and AB are normal. | 5 | CO5 | K3 |