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GANDHI INSTITUTE OF ENGINEERING AND TECHNOLOGY UNIVERSITY, ODISHA, GUNUPUR
(GIET UNIVERSITY)

M. Sc. (Third Semester) Regular Examinations, December – 2024

22MTPC301 – Measure Theory and Integration

(M.Sc. Mathematics)

Time: 3 hrs | Maximum: 70 Marks

(The figures in the right hand margin indicate marks.)

PART – A **(2 x 10 = 20 Marks)**

Q.1. Answer ***ALL*** questions

	Question	CO #	Blooms Level
a.	Define field.	CO1	K1
b.	Define Countable additative measure over countable disjoint union set.	CO1	K2
c.	Prove that A is countable outer measure of A is zero	CO1	K3
d.	Prove that R is measurable set	CO2	K3
e.	Define Simple function of bounded set of finite measure	CO2	K2
f.	Define Chebychev's Inequality	CO2	K1
g.	Explain absolutely continuous functions.	CO3	K1
h.	Define Jordan's Theorem.	CO3	K1
i.	Define The Riesz-Fischer Theorem	CO4	K2
j.	State Holder Inequlity Theorem.	CO4	K1

PART – B **(10 x 5 = 50 Marks)**

Answer ***ANY FIVE*** questions

	Question	Marks	CO #	Blooms Level
2. a.	Prove that Union of two measurable sets is measurable.	5	CO1	K2
b.	The interval (a,∞)is measurable.	5	CO1	K2
3.a.	Let A be any set, and $E_1,\ldots,E_n$ a finite sequence of disjoin measurable sets. Then $m^*(A\cap[\bigcup_{i=1}^{n}E_i]) = \sum_{i=1}^{n} m^*(A\cap E_i)$.	7	CO1	K2
b.	Prove that If $m^*A = 0,\ then\ m^*(A\cup B) = m^*B$	3	CO1	K2
4.a.	Let $\{A_n\}$ be a countable collection of sets of real numbers. Then $m^*(\cup A_n) \le \sum m^*A_n$.	5	CO2	K2
b.	Let α be a constant and $f\ and\ g$ two measurable real- valued functions defined on the same domain. Then the functions i. αf is a measurable on E ii. $\alpha f + \beta g$ is a measurable on E. iii. fg is a measurable on E	5	CO2	K2
5.a.	If $f\ and\ g$ are nonnegative measurable function, then: i. $\int_E \alpha f = \alpha\int_E f\quad \alpha > 0$. ii. $\int_E(\alpha f + bg) = \alpha\int_E f + \beta\int_E g$. iii. $If\ f \le g\ then\ \int_E f \le \int_E g$	5	CO2	K3

b.	Show that if $f(x) = c$, then for every partition P of [a, b], $U(P,f) = L(P,f) = c(b-a)$ Hence f is Riemann integrable a nd $\int_a^b f(x)dx = c(b-a)$.	5	CO2	K3
6.a	Let f be defined and bounded on a measurable set E with mE finite. In order that $\inf_{f\le\psi}\int_E \psi(x)\,dx = \sup_{f\ge\varphi}\int_E \varphi(x)\,dx$ For all simple function $\varphi$and ψ, I is necessary and sufficient that f be measurable	5	CO3	K2
b.	State and prove Fatou's Lemma	5	CO3	K2
7.	If the function f is monotone on the open interval [a, b]. Then f is differentiable almost everywhere. The derivative f' is measurable, and $\int_a^b f(x)\,dx \le f(a) - f(b)$	10	CO3	K2
8 a.	State and prove the Minkowski Inequality	5	CO4	K2
b.	If p and q are nonnegative extended real number such that $\frac{1}{p}+\frac{1}{q} = 1$ and If $f \in L^p$ and $g \in L^p, then$ $f.g \in L^1$ and $\int \lvert f.g\rvert = \lvert\lvert f\rvert\rvert_p . \lvert\lvert g\rvert\rvert_p$ Equality holds if and only if for some constant α and β, not both are zero, we have $\alpha\lvert f\rvert^p = \beta\lvert g\rvert^q$	5	CO4	K3

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