Reg. No | | | | | | | | | | | AY 24

GANDHI INSTITUTE OF ENGINEERING AND TECHNOLOGY, ODISHA, GUNUPUR
(GIET UNIVERSITY)

M.Sc. (First Semester - Regular) Examinations, February – 2025

24MPCMA11002 – Real Analysis

(Mathematics)

Time: 3 hrs — Maximum: 60 Marks

Answer ALL questions

(The figures in the right hand margin indicate marks)

PART – A **(2 x 5 = 10 Marks)**

Q.1. Answer ***ALL*** questions

		CO #	Blooms Level
a.	Is every bounded sequence convergent? Justify your answer.	CO1	K1
b.	The series $\sum_{n=1}^{\infty}(1+\frac{1}{n})$ is convergent or divergent.	CO2	K2
c.	Examine the convergence of $\int_0^1 \frac{dx}{\sqrt{(1-x)}}$.	CO2	K2
d.	Find the pointwise convergence of sequence of function $f_n(x) = \frac{x^{2n}}{1+x^{2n}}$.	CO2	K2
e.	Define Directional Derivative.	CO1	K1

PART – B **(10 x 5 = 50 Marks)**

Answer ***ALL*** the questions

		Marks	CO #	Blooms Level
2. a.	State and Prove Cauchy Completeness Principle Theorem.	5	CO2	K1
b.	Test $\sum_{n=1}^{\infty}\frac{\sin nx}{n^2}$ is absolute convergent.	5	CO1	K1
	(OR)			
c.	Prove that the series $\sum\frac{1}{n!}$= 1+1+$\frac{1}{2!}$+$\frac{1}{3!}$+---------+$\frac{1}{n!}$+---- convergent to the value e.	10	CO1	K2
3.a.	State and Prove Intermediate Value Theorem.	5	CO2	K1
b.	Show that the function f(x)=1/x is not uniformly continuous on]0,1].	5	CO2	K2
	(OR)			
c.	A function f is defined on R by $$f(x)=\begin{cases} -x^2 & if\ x \le 0 \\ 5x-4 & \text{if } 0 < x \le 1 \\ 4x^2-3x & \text{if } 1 < x < 2 \\ 3x+4 & \text{if } x \ge 2 \end{cases}$$ Examine f for continuity at x=0,1,2. Also discuss the kind of discontinuity, if any.	7	CO3	K3
d.	Let $(x_n),(y_n)$ be two sequences. Define two other sequences by M_n=max $\{x_n,y_n\}, \forall n \in z^+$.suppose $\log_{n\to\infty} x_n$ *and* $\log_{n\to\infty} y_n$ both exist. Then shows that both lim $M_n$and lim m_n exist.	3	CO3	K3
4.a	For what value of m, n is the integral $\int_0^1 x^{m-1}(1-x)^{n-1}\log x\, dx$ convergent.	10	CO4	K2
	(OR)			
b.	Show that, a)$\int_0^1 \frac{\log x}{\sqrt{x}}$ dx is convergent, but b)$\int_1^2 \frac{\sqrt{x}}{\log x}$ dx is divergent.	10	CO4	K2
5.a.	Test for uniform convergence, the sequence <f_n(x)>, where $f_n(x)=1-\frac{x^n}{n}$, [0,1].	10	CO3	K2

	(OR)			
b.	State and Prove Weierstrass Approximation Theorem.	10	CO2	K1
6.a.	State and Prove Taylor Theorem.	10	CO2	K1
	(OR)			
b.	Find $\frac{dy}{dx}$ when y^{x^y}=sin x.	5	CO5	K3
c.	Find $\frac{\partial w}{\partial \theta}$ and $\frac{\partial w}{\partial \emptyset}$ given that w(x,y,z)=f($x^2 + y^2 + z^2$) where x=r cosθ.cos$\emptyset$,y= r cosθ.sin$\emptyset$,z= r sinθ.	5	CO5	K3

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