GANDHI INSTITUTE OF ENGINEERING AND TECHNOLOGY, ODISHA, GUNUPUR

(GIET UNIVERSITY)

M. Sc. (First Semester - Regular) Examinations, February – 2025

24MPCMA11004 -Ordinary Differential Equations

Time: 3 hrs Maximum: 60 Marks

Answer ALL questions

(The figures in the right hand margin indicate marks)

PART – A **(2 x 5 = 10 Marks)**

Q.1. Answer ***ALL*** the questions		CO #	Blooms Level
a.	Define Lipschitz condition for function of two variables.	CO1	K1
b.	Are the functions e^x, $\sin x$, $\cos x$ linearly independent or dependent? Justify your answer.	CO2	K2
c.	Find $P_5(x)$ by Rodrigues formula.	CO2	K2
d.	Define Analytic function. Give an example of singular and regular points of a differential equation.	CO3	K1
e.	When the second order differential equation $x'' = f(t, x, x')$ is called oscillatory and non-oscillatory.	CO4	K2

PART – B **(10 x 5 = 50 Marks)**

Answer ***ALL*** the questions		Marks	CO #	Blooms Level
2.a.	State the Picard's theorem for existence and uniqueness of solution by Picard's successive approximation method.	3	CO1	K3
b.	Find the solution of following IVP $\acute{x} = -x(t)$; $x(0) = 1$, $t > 0$ by Picard's succussive approximation.	7	CO1	K3
	(OR)			
c.	Is $f(t, x) = x^{1/2}$ satisfy Lipschitz condition on R. (i) $= \{(t, x): \|t\| \leq 2, \|x\| \leq 2\}$ (ii) $= \{(t, x): \|t\| \leq 2, 2 \leq \|x\| \leq 4\}$.	5	CO1	K3
d.	Prove that, let $I \subset$R be an interval. A continuous function $x: I \to R$ is a solution of IVP $x\acute{(}t) = f(t, x(t))$; $x(t_0) = x_0$ on interval I if and only if x is a solution of $x(t) = x_0 + \int_{t_0}^{t} f(s, x(s))ds$.	5	CO1	K3
3.a.	Show that the functions $x_1(t) = t^2, x_2(t) = t\|t\|, -\infty < t < \infty$ are linearly independent. Also show that their Wronskian is zero.	5	CO2	K2
b.	Solve $x^2y'' + 0.6xy' + 16.04y = 0$.	5	CO2	K2
	(OR)			
c.	Solve $y'' + 2y' - 35y = 12e^{5x} + 37 \sin 5x$ by method of undetermined coefficients.	7	CO2	K2
d.	Solve $y'' + 6y' + 5y = 0$.	3	CO2	K2
4.a.	Find the power series solution of $(1 - x^2)y'' - 2xy' + 20y = 0$ in powers of x.	7	CO 3	K3
b.	What are the singular and ordinary points of differential equation $(1 - x^2)y'' - 2xy' + 20y = 0$.	3	CO3	K3
	(OR)			
c.	Find the power series solution of $y'' - 4xy' + (4x^2 - 2)y = 0$ in powers of x.	7	CO3	K3
d.	Find the power series solution of $y' + y = 0$ in powers of x.	3	CO3	K3
5.a.	Prove that Strum's comparison theorem. That is, let r_1,r_2 and p be continuous	10	CO4	K4

functions on (a, b) and $p > 0$. Assume that x and y are real solutions of $(px')' + r_1 x = 0$ and $(py')' + r_2 y = 0$ respectively on (a, b). If $r_2(t) > r_1(t)$, for $t \in (a, b)$ then between any two consecutive zeros t_1,t_2 of $x \in (a, b)$, there exist a non-zero of y in $[t_1, t_2]$. Moreover, when $r_1 \equiv r_2$ in $[t_1, t_2]$, the conclusion still holds if $x$and y are linearly independent.

(OR)

b.	Find the solution for Legendre's equation $(1 - x^2)y'' - 2xy' + 12y = 0$.	7	CO4	K4
c.	State Strum's separation theorem.	3	CO4	K2
6.a.	Define (i) Green's Function and (ii) Prove that Green's function is symmetric.	10	CO5	K3
	(OR)			
b.	Solve the IVP $8y'' - 6y' + y = 6\ \cos hx, y(0) = 0.2$ and $y'(0) = 0.05$.	10	CO5	K3

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