QP Code: Reg. No [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ] AY 24

GANDHI INSTITUTE OF ENGINEERING AND TECHNOLOGY, ODISHA, GUNUPUR
(GIET UNIVERSITY)

M.Sc. (First Semester - Regular) Examinations, February – 2025

24MPCMA11001 – Linear Algebra
(Mathematics)

Time: 3 hrs Maximum: 60 Marks

Answer ALL questions
(The figures in the right hand margin indicate marks)

PART – A **(2 x 5 = 10 Marks)**

Q.1. Answer ***ALL*** questions		CO #	Blooms Level
a.	Show that A^{-1} exists if 0 is not an eigen value of A.	CO3	K1
b.	Define inner product space and give an example.	CO4	K1
c.	Write a matrix and verify the Caley-Hamilton Theorem.	CO2	K2
d.	Write the statement of the Rank-Nullity theorem.	CO1	K1
e.	Define dual space with example	CO5	K2

PART – B **(10 x 5 = 50 Marks)**

Answer ***ALL*** the questions		Marks	CO #	Blooms Level
2. a.	If U and W are subspaces of a vector space V such that $U \subset W$, then prove that U is a subspace of W.	6	CO1	K1
b.	Check, whether the set is $\{(1,1,1),(1,-1,1),(3,-1,3)\}$ linearly independent or dependent.	4	CO1	K2
	(OR)			
c.	Define linear transformation.	2	CO1	K1
d.	Let $T: P_4 \to P_4$ be defined by $T(p)(x) = \int_1^x p'(t)\,dt$. Let $B_1 = B_2 = \{1, x, x^2, x^3, x^4\}$ be the basis for P_4. Find the matrix of T relative to bases B_1 and B_2.	8	CO1	K3
3.a.	Define rank of matrix. Write its use	3	CO2	K1
b.	Determine the values of a and b for which the system $x + 2y + 3z = 6$, $x + 3y + 5z = 9$, $2x + 5y + az = b$ has (i) no solution (ii) unique solution (iii) infinite number of solutions.	7	CO2	K2
	(OR)			
c.	Define LU-decomposition.	2	CO2	K1
d.	Find the LU-decomposition of the matrix $A = \begin{bmatrix} 2 & -1 & -1 \\ 0 & -4 & 2 \\ 6 & -3 & 1 \end{bmatrix}$.	8	CO2	K2
4.a.	Determine the nature, index and signature of the quadratic form $2x_1^2 + 2x_2^2 + 3x_3^2 + 2x_1x_2 - 4x_1x_3 - 4x_2x_3$.	5	CO3	K2
b.	Show that Transpose of a unitary matrix is unitary.	5	CO3	K1
	(OR)			
c.	Write on diagonalization of a matrix.	2	CO3	K1

d.	Find the modal matrix and diagonalizes the matrix $C = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ -4 & 4 & 3 \end{bmatrix}$.	8	CO3	K3
5.a.	Assume that $p(t)$ is a minimal polynomial of a linear operator T on a finite dimensional vector space V. Show that if $g(T) = 0$, then $p(t)$ divides $g(t)$, for any polynomial $g(t)$ then the minimal polynomial $p(t)$ divides the characteristic polynomial of T.	6	CO4	K3
b.	Find the minimal polynomial of the matrix $A = \begin{bmatrix} 3 & -1 & 0 \\ 0 & 2 & 0 \\ 1 & -1 & 2 \end{bmatrix}$.	4	CO4	K2
	(OR)			
c.	If $T \in A(V)$ has all its characteristic roots in F, then there is a basis of V in which the matrix of T is triangular.	8	CO4	K2
d.	Explain minimal polynomial. Give an example.	2	CO4	K1
6.a.	Write on Gram-Schmidt Orthogonalization Process.	3	CO4	K1
b.	Write the derivation for Gram-Schmidt Orthogonalization Process.	7	CO4	K2
	(OR)			
c.	Define Algebra of Linear transformation.	2	CO5	K1
d.	If V and W are of dimensions m and n respectively over F then $\text{Hom}(V, W)$ is of dimension mn over F.	8	CO5	K2

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